Beam It Up!
— A Classification Grid for Historic and Contemporary Practices of Beaming by Mathematical Re-Modelling

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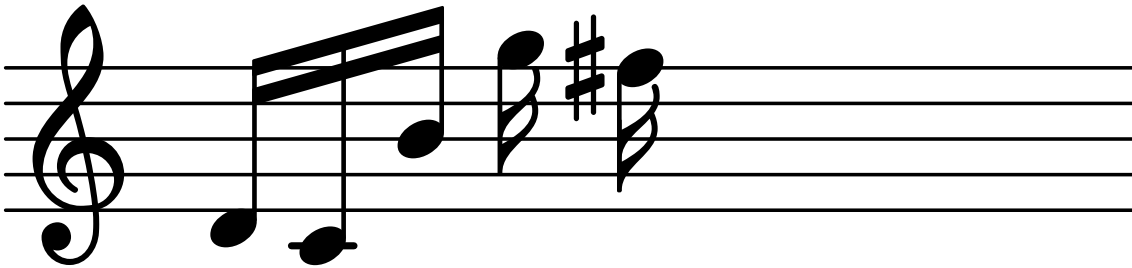
Tenor 2023, Boston, May 17 2023



1. **Context**
   - Genesis of the earliest beams
   - Sort it out by mathematical re-modelling

2. **Evaluation Pipeline for Beams**
   - Overview
   - Metric Tree Spec and Rhythms
   - Additional External Parameters
   - Pitches and Vertical Positions
   - Bypass a): Modified Genuine Beams

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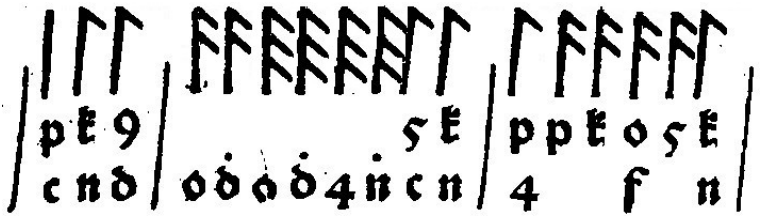

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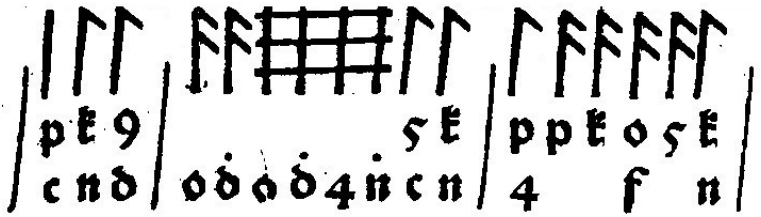

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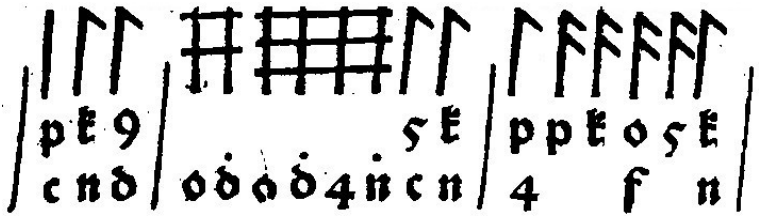

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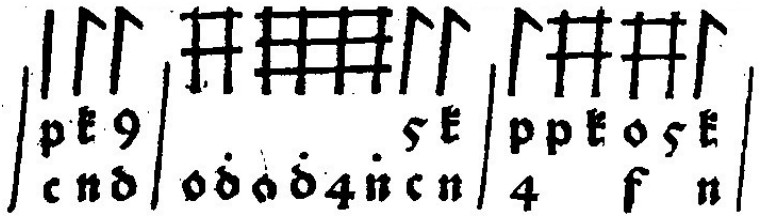

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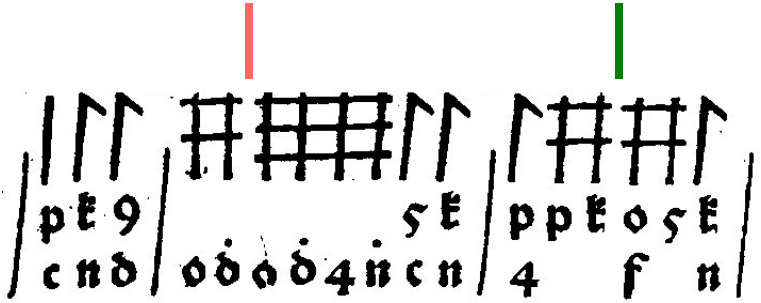

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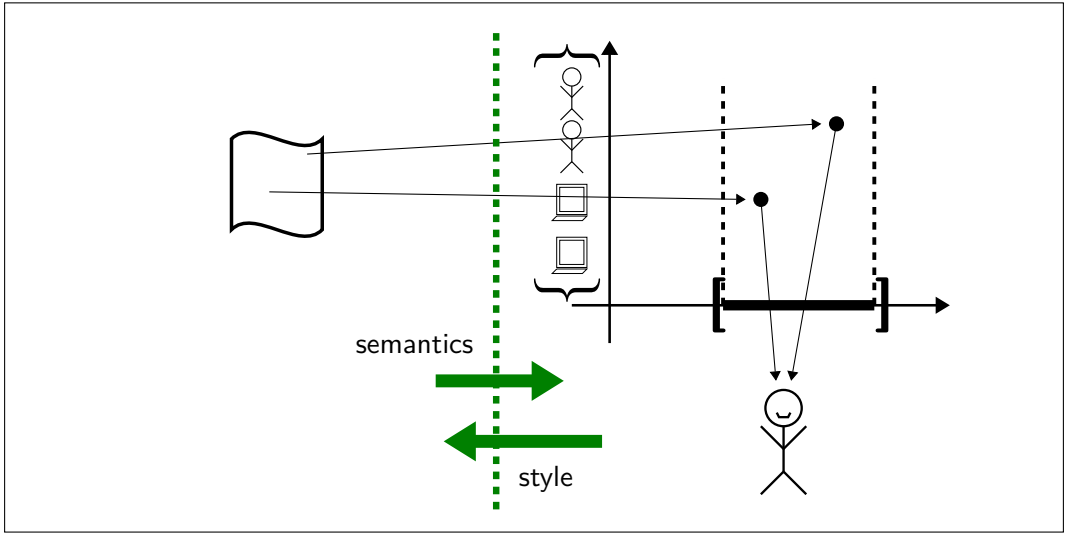
Mathematical re-modelling =

- Build a mathematical model which mimicks the symbol system defined by culture / history
- Build *variants* / *families* of models, identified by *parameter settings*
- With **no claim** for explanation, historic truth, adequateness,
  but as **an offer** to domain experts for more precision in nomenclature.

Basic execution model of musical notation



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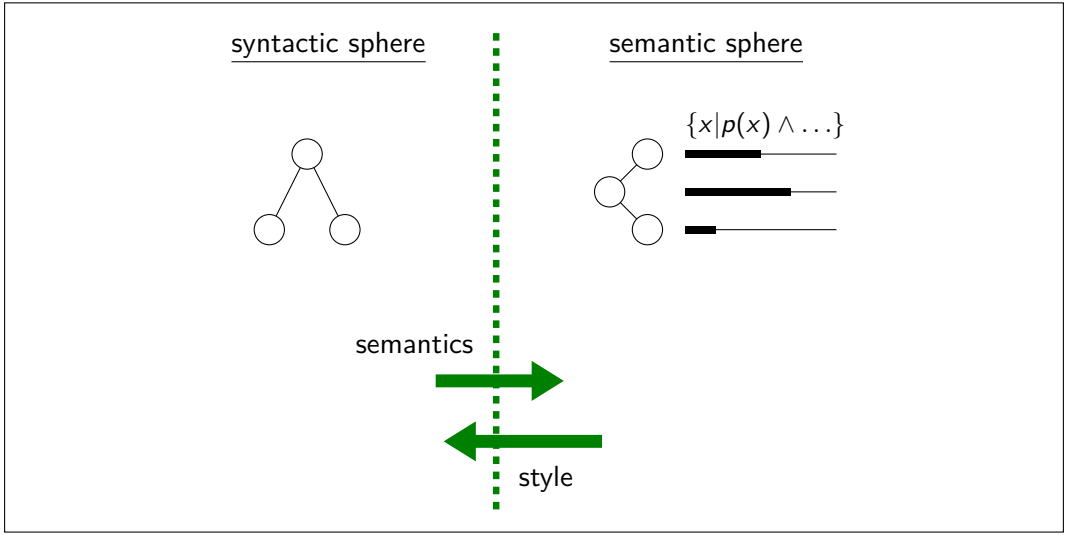
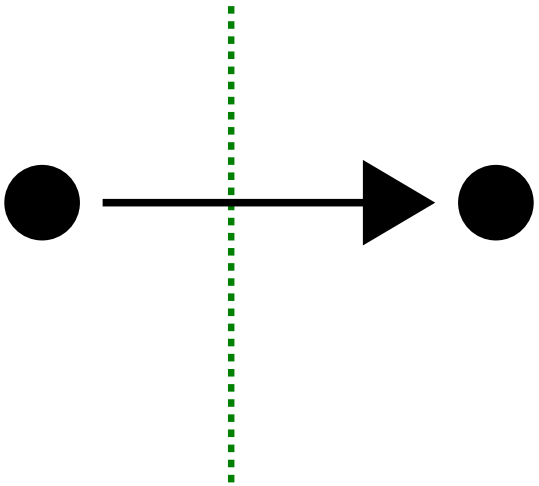


Basic execution model of musical notation

reading a score:



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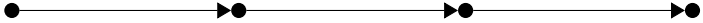
reading a score by evaluating a pipeline:



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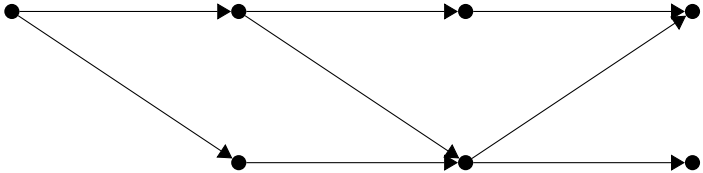

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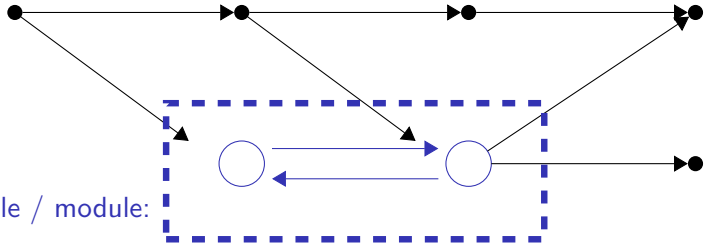

quadruple / module:

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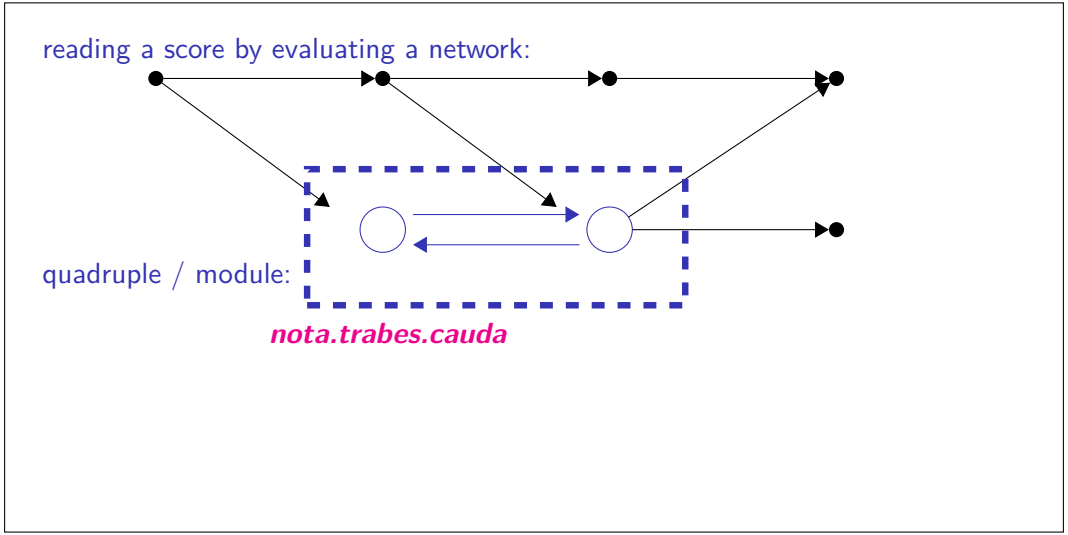

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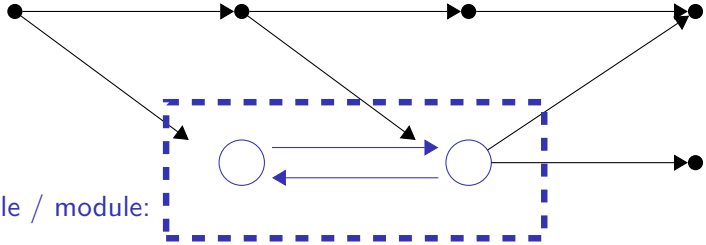

quadruple / module:

nota.trabes.cauda

names and types of parameters:

nota.trabes.cauda.minInterior : $\mathbb{Q}_{\geq 0}$

nota.trabes.cauda.min : $\mathbb{Q}_{\geq 0}$

$\Longrightarrow$ model properties

de Linguis Musicam Notare (LMN)

- Complete theory of musical notation
  restricted to *conventional use of CWN*
- 834 properties (= modules and their parameters)
- 700 pages
- **open and extensible**
  by "Z" refinement mechanism and hierarchical nomenclature
- These results on beams are such an extension to LMN: about 60 properties

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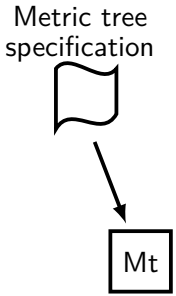

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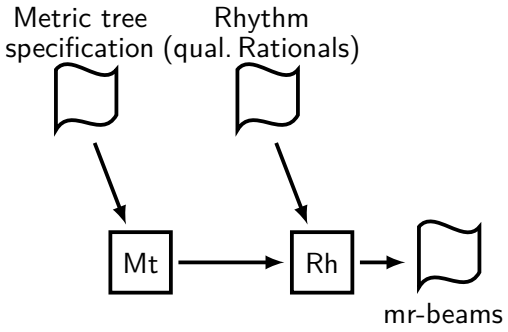

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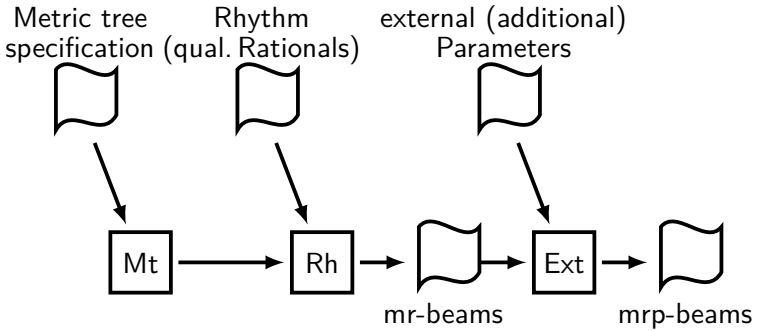

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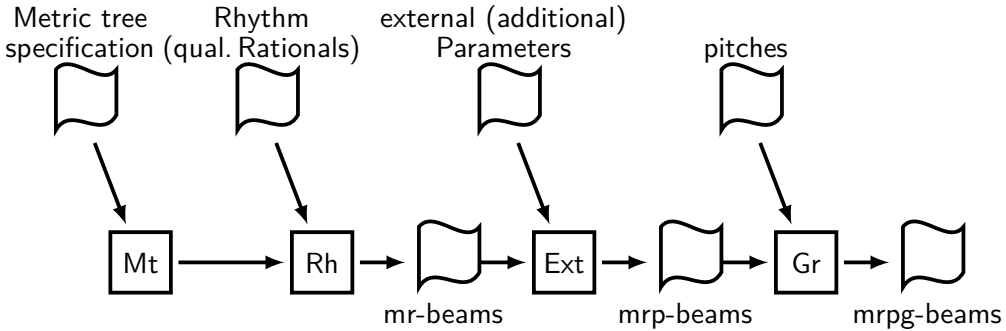

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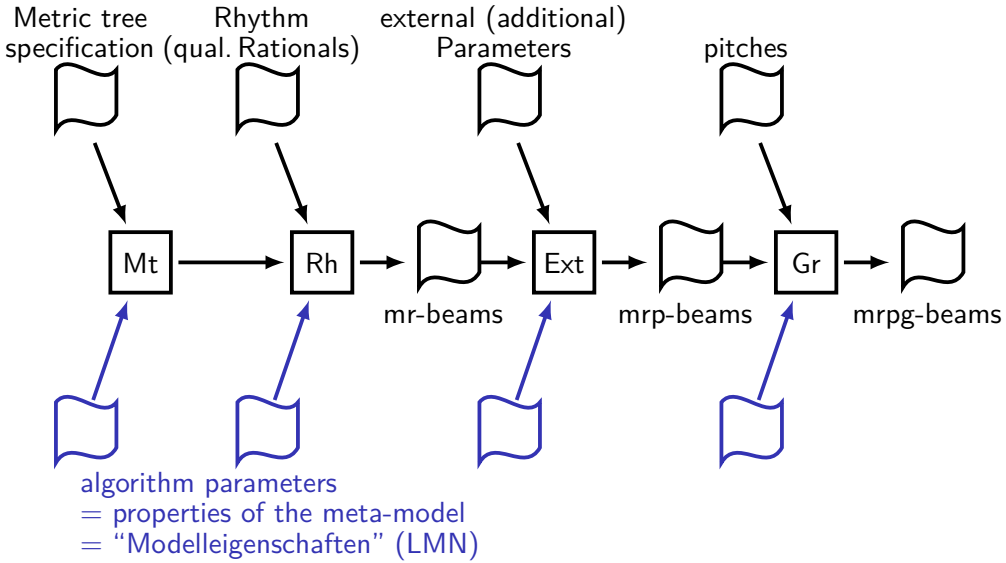

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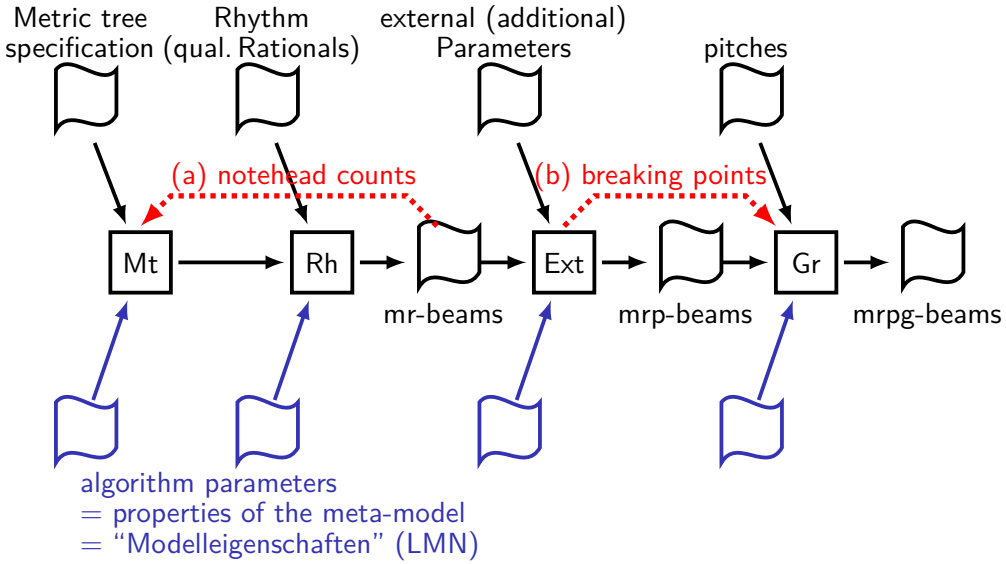
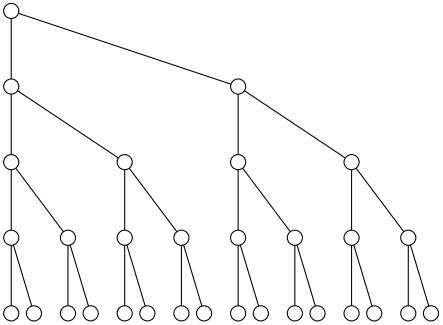
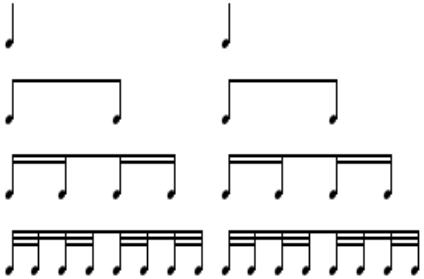


Metric Tree and Rhythm:

`MTSpec = 1/2` (according to "metricSplit", Journal of Mathematics and Music, 2019)



Genuine beams:



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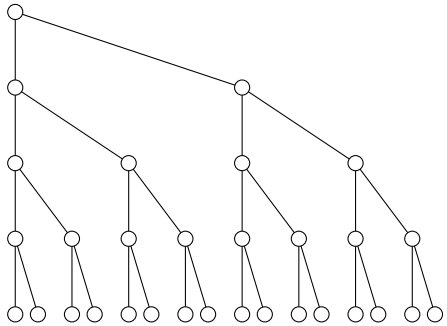

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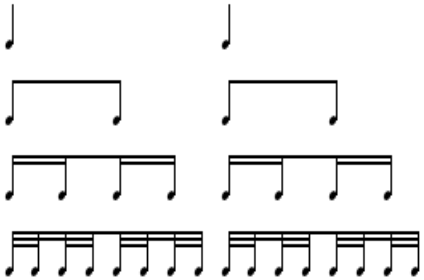

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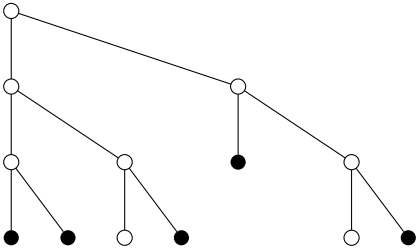

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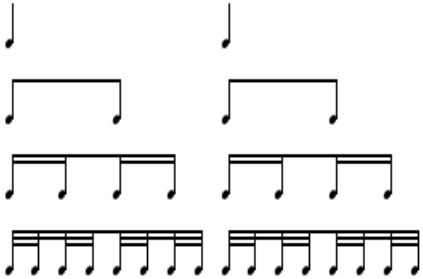

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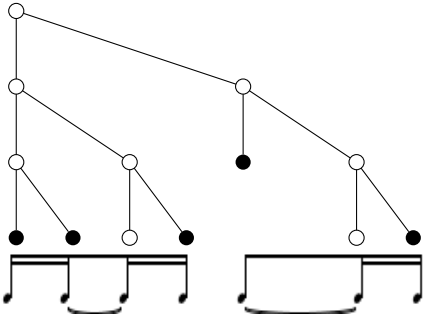

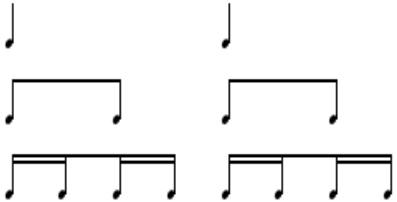

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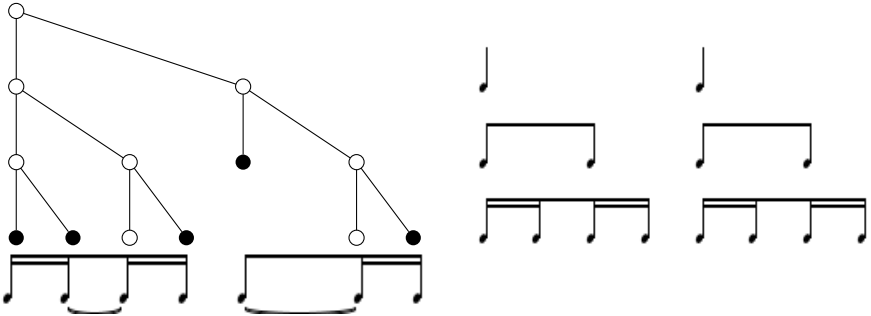

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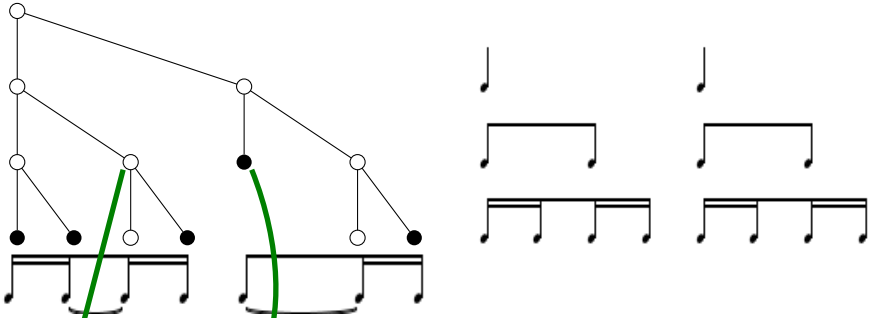

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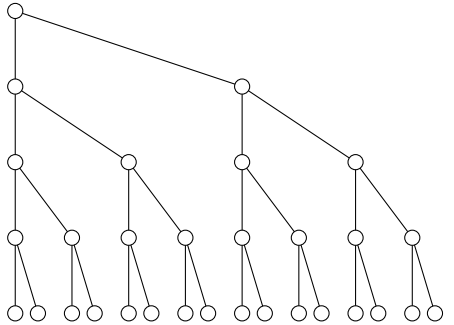

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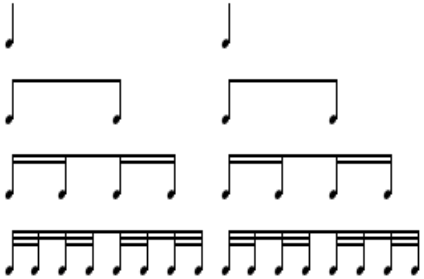

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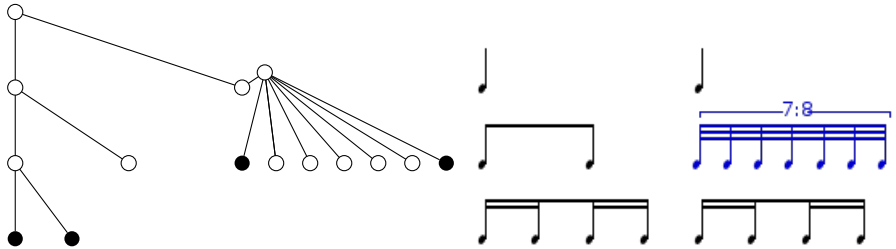

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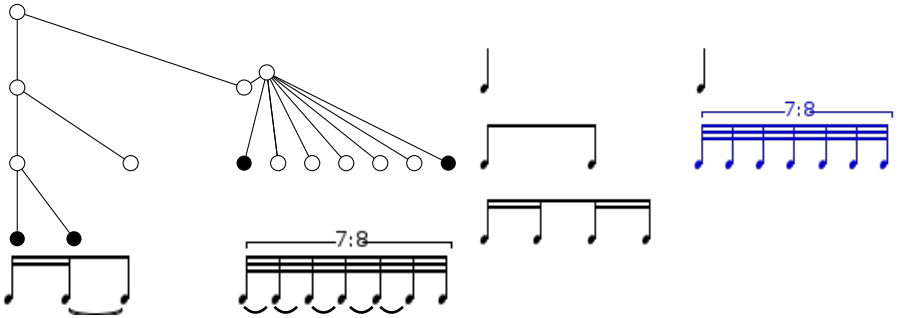

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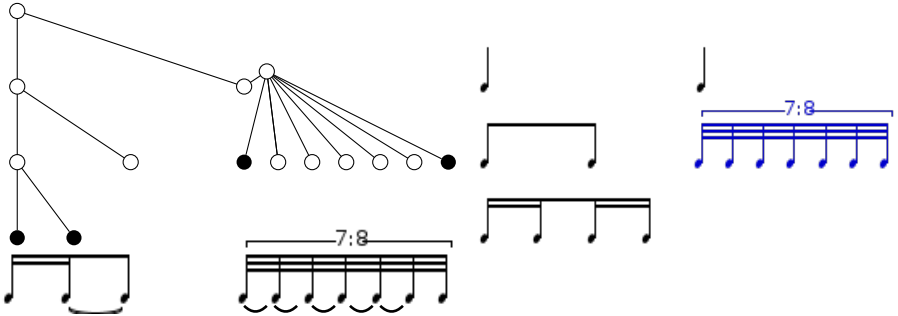

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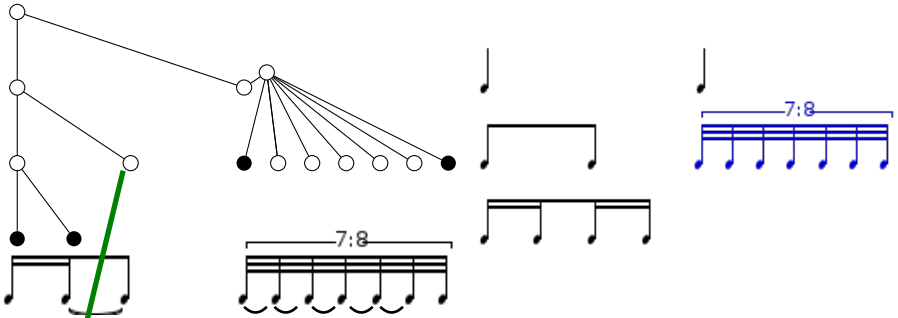

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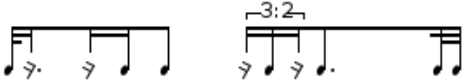
nota.trabes.superPausam.perCaudulum



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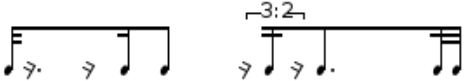
nota.trabes.superPausam.transiens



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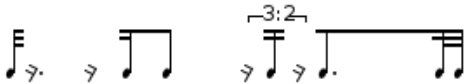

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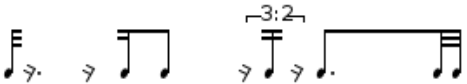
nota.trabes.superPausam.ELIM-trabulaeOmnesContraTrabes



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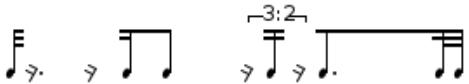

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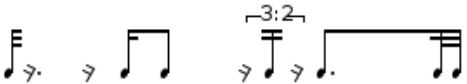

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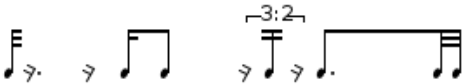
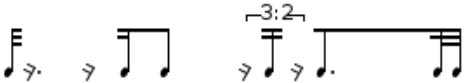
nota.trabes.superPausam.ELIM-trabulaNonSubTrabem
..-ELIM-TrabulaeOmnesContraTrabes & ..ELIM-trabulaNonSubTrabem



Pauses and Generation and Cancellation of Beamlets:

nota.trabes.superPausam.perCaudulum
nota.trabes.superPausam.transiens
nota.trabes.superPausam.separans

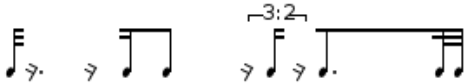
nota.trabes.superPausam.ELIM-trabulaeOmnesContraTrabes
nota.trabes.superPausam.ELIM-trabulaNonSubTrabem
..-ELIM-TrabulaeOmnesContraTrabes & ..ELIM-trabulaNonSubTrabem



Pauses and Generation and Cancellation of Beamlets:

nota.trabes.superPausam.perCaudulum
nota.trabes.superPausam.transiens
nota.trabes.superPausam.separans

nota.trabes.superPausam.ELIM-trabulaeOmnesContraTrabes
nota.trabes.superPausam.ELIM-trabulaNonSubTrabem
..-ELIM-TrabulaeOmnesContraTrabes & ..ELIM-trabulaNonSubTrabem
nota.trabes.superPausam.ELIM-trabulaeOmnesContraIdem(dexter)



Pauses and Generation and Cancellation of Beamlets:

nota.trabes.superPausam.perCaudulum

nota.trabes.superPausam.transiens

nota.trabes.superPausam.separans

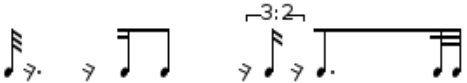
nota.trabes.superPausam.ELIM-trabulaeOmnesContraTrabes

nota.trabes.superPausam.ELIM-trabulaNonSubTrabem

..-ELIM-TrabulaeOmnesContraTrabes & ..ELIM-trabulaNonSubTrabem

nota.trabes.superPausam.ELIM-trabulaeOmnesContraIdem(dexter)

nota.trabes.superPausam.ELIM-trabulaSola



Pauses and Generation and Cancellation of Beamlets:

$$(\mathit{flags} \mid \mathit{leftLong}, \mathit{leftShort} \mid \mathit{rightLong}, \mathit{rightShort}) : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N}$$

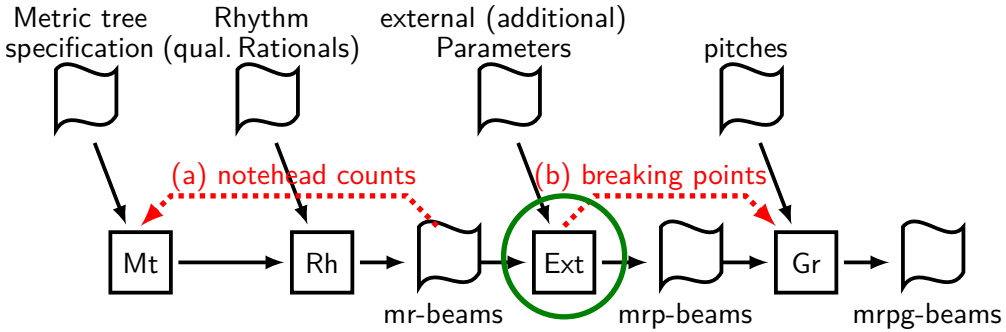
$$\frac{b > 0}{\begin{array}{c}\mathit{ELIM\text{-}trabulaeContraIdem}((0 \mid a, b \mid a, b), \mathit{dexter}) = (0 \mid a, 0 \mid a, b) \\ \mathit{ELIM\text{-}trabulaeContraIdem}((0 \mid a, b \mid a, b), \mathit{sinister}) = (0 \mid a, b \mid a, 0)\end{array}}$$

$$\frac{b > 0 \qquad a + b \leq c}{\begin{array}{c}\mathit{ELIM\text{-}trabulaeOmnesContraTrabes}(0 \mid a, b \mid c, d) = (0 \mid a, 0 \mid c, d) \\ \mathit{ELIM\text{-}trabulaeOmnesContraTrabes}(0 \mid c, d \mid a, b) = (0 \mid c, d \mid a, 0)\end{array}}$$

$$\frac{b > 0 \qquad c > 0 \qquad x = \mathrm{MAX}(d, b - c)}{\begin{array}{c}\mathit{ELIM\text{-}trabulaNonSubTrabem}(0 \mid 0, b \mid c, d) = (0 \mid 0, 0 \mid c, x) \\ \mathit{ELIM\text{-}trabulaNonSubTrabem}(0 \mid c, d \mid 0, b) = (0 \mid c, x \mid 0, 0)\end{array}}$$

$$\frac{b + d > 0}{\mathit{ELIM\text{-}trabulaSola}(0 \mid 0, b \mid 0, d) = (\mathrm{MAX}(b, d) \mid 0, 0 \mid 0, 0)}$$

Additional External Parameters



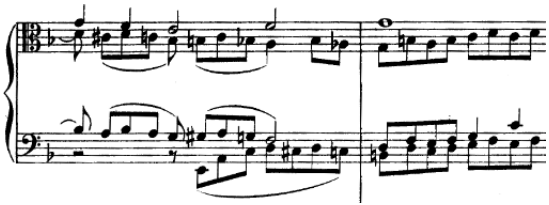
Additional External Parameters

nota.trabes.cumVerborum.syllabis/nominibus/lineis[Trancón]

Additional External Parameters

nota.trabes.cumVerborum.syllabis/nominibus/lineis[Trancón]

nota.trabes.separate.cumMelo/cumLegato



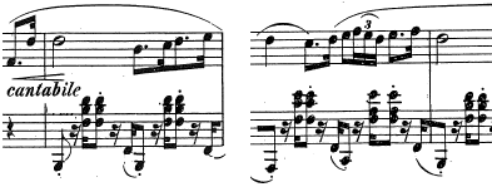
Additional External Parameters

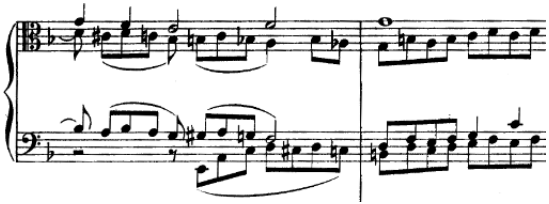
nota.trabes.extera.separans/ligans

nota.trabes.cumVerborum.syllabis/nominibus/lineis[Trancón]

nota.trabes.separate.cumMelo/cumLegato

nota.trabes.cumPositioneManus (∈*ligans*!)





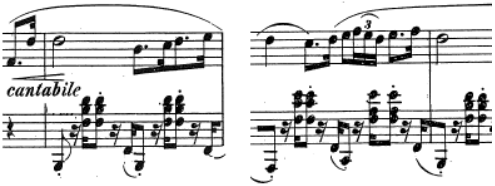
Additional External Parameters

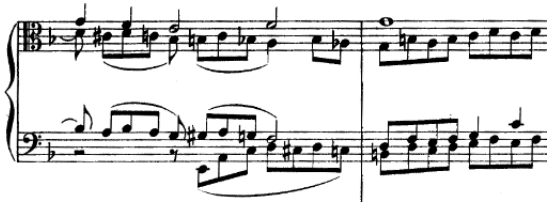
nota.trabes.extera.separans/ligans

nota.trabes.cumVerborum.syllabis/nominibus/lineis[Trancón]

nota.trabes.separate.cumMelo/cumLegato

nota.trabes.cumPositioneManus (∈*ligans*!)

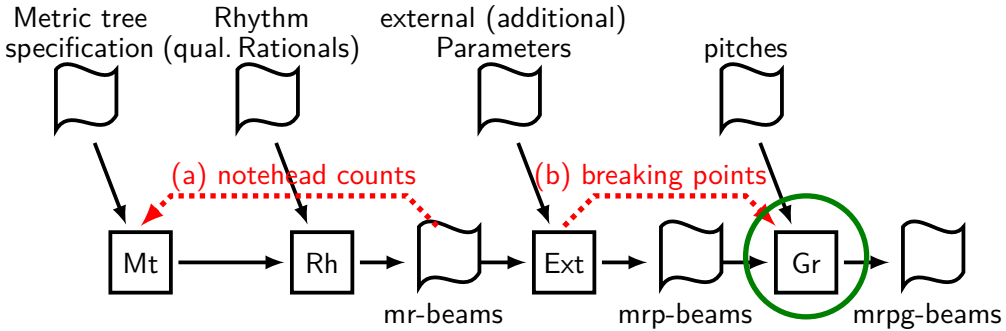




nota.trabes.accelerans : $(\mathbb{Q} \times \mathbb{N}_{>0} \times \mathbb{Q})^*$

$= \langle (0, 30, 1/1)(1/4, 90, 3/2), (1/4, 60, 1/1), (1/2, 60, 1/1), (1/2, 30, 1/2), (1/1, 120, 3/2) \rangle$

Pitches and Vertical Positions



Pitches and Vertical Positions

Vertical arrangement, according to pitches and graphical constraints, may ...

- be trivial

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- or need breaking into sub-aggregates

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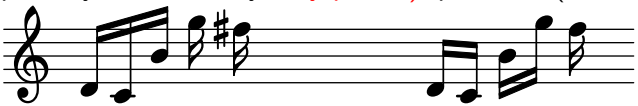
Vertical arrangement, according to pitches and graphical constraints, may . . .

- be trivial
- or need breaking into sub-aggregates
- possibly controlled by a by-pass b): penalties (“Sollbruchstellen”) :

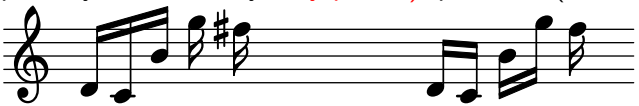


(a) Flute (b) Piano (*cumPositioneManus*)

Pitches and Vertical Positions

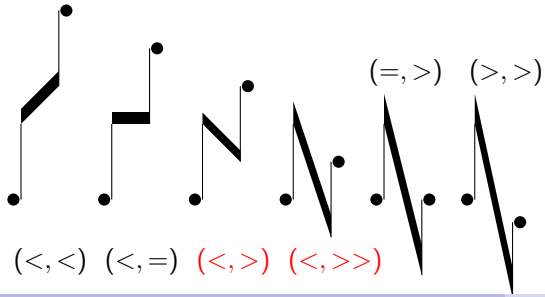
Vertical arrangement, according to pitches and graphical constraints, may ...

- be trivial
- or need breaking into sub-aggregates
- possibly controlled by a by-pass b): penalties ("Sollbruchstellen") :



(a) Flute (b) Piano (*cumPositioneManus*)

- or require to switch the beam side, forming a "knee" (*genus.inLineae/interLineas*):

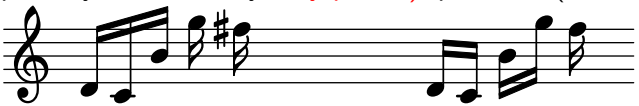


nota.trabes.genuParadoxum
(stands against ... *inclinationSignificans*)

Pitches and Vertical Positions

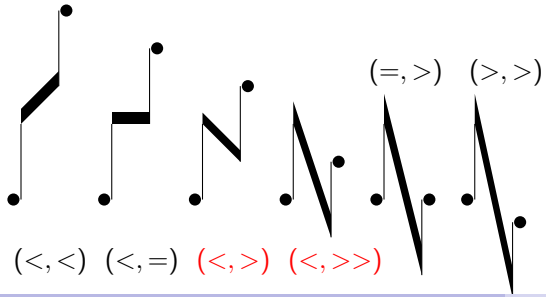
Vertical arrangement, according to pitches and graphical constraints, may ...

- be trivial
- or need breaking into sub-aggregates
- possibly controlled by a by-pass b): penalties ("Sollbruchstellen") :



(a)Flute (b) Piano (*cumPositioneManus*)

- or require to switch the beam side, forming a "knee" (*genus.inLineae/interLineas*):

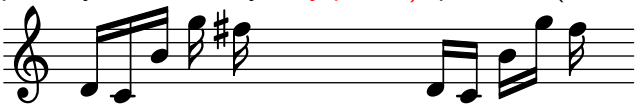


nota.trabes.genuParadoxum
(stands against ... *inclinationSignificans*)

Pitches and Vertical Positions

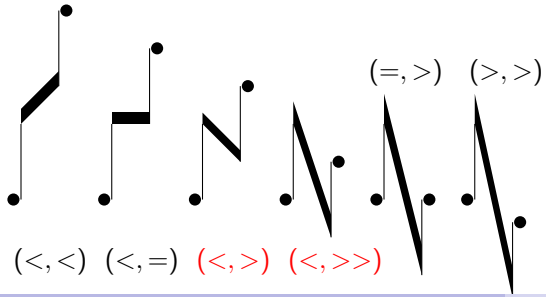
Vertical arrangement, according to pitches and graphical constraints, may ...

- be trivial
- or need breaking into sub-aggregates
- possibly controlled by a by-pass b): penalties ("Sollbruchstellen") :



(a)Flute (b) Piano (*cumPositioneManus*)

- or require to switch the beam side, forming a "knee" (*genus.inLineae/interLineas*):

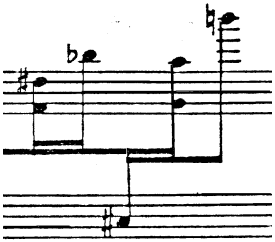


nota.trabes.numerusGenuum: $\mathbb{N}_0$
nota.trabes.numerusInGenui: $\mathbb{N}_1$
nota.trabes.numerusInterGenibus: $\mathbb{N}_1$
nota.trabes.caudaVersusCaudam: bool

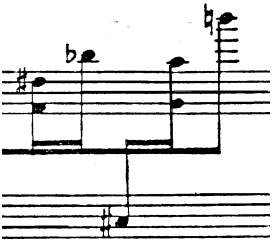
nota.trabes.genuParadoxum
(stands against ... *inclinationSignificans*)

More Details on Knees:

nota.trabes.summaeInGenu



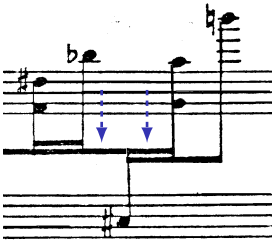
nota.trabes.inGenuCumPluribus



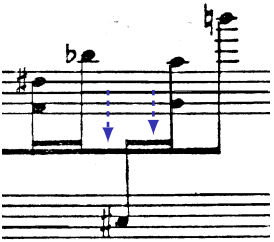
(Original Xenakis, Herma)

More Details on Knees:

nota.trabes.summaeInGenu



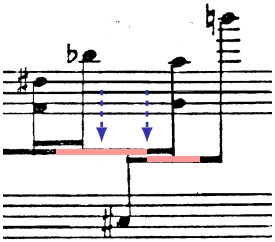
nota.trabes.inGenuCumPluribus



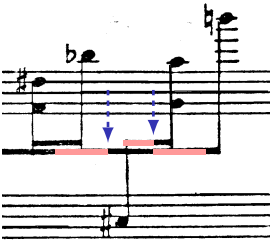
(Original Xenakis, Herma)

More Details on Knees:

nota.trabes.summaeInGenu



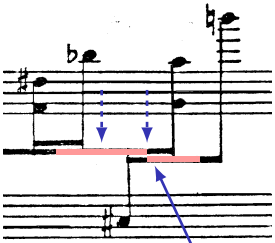
nota.trabes.inGenuCumPluribus



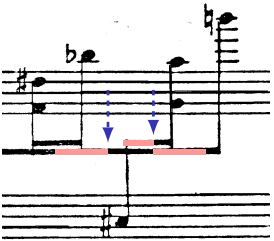
(Original Xenakis, Herma)

More Details on Knees:

nota.trabes.summaeInGenu



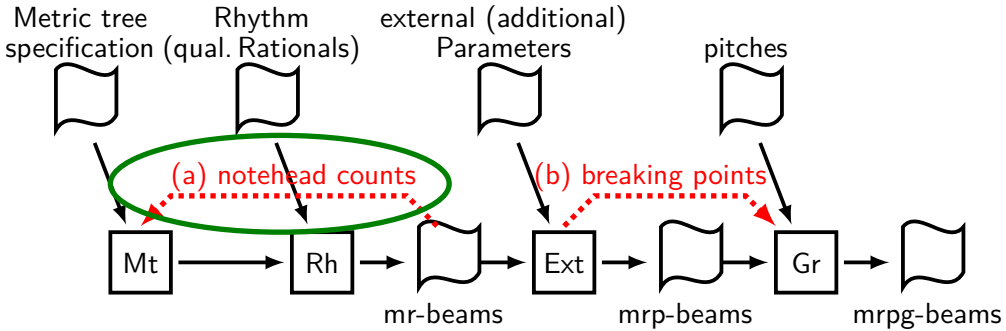
nota.trabes.inGenuCumPluribus



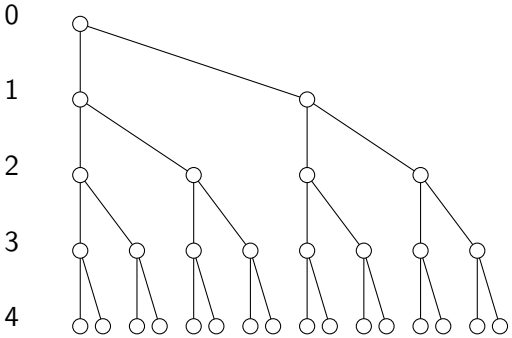
(Original Xenakis, Herma)

Change of meaning is meaningless / non-deterministic determinedness
= *infra.nonDeterminataSedDeterminans[Pepper]*

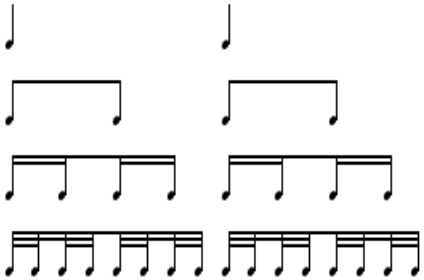
Bypass a): Modified Genuine Beams



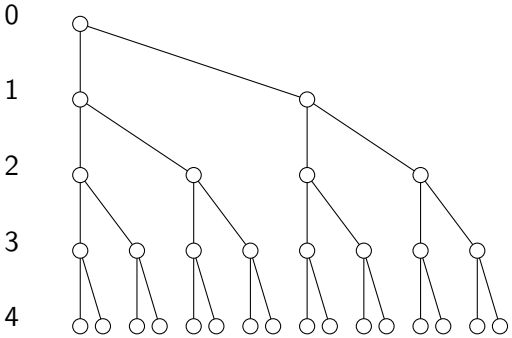
Bypass a): Modified Genuine Beams



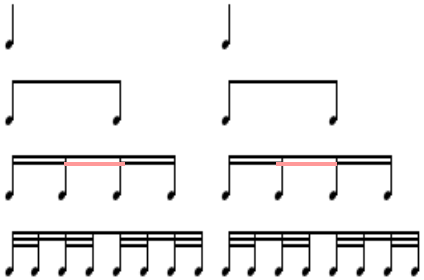
Genuine beams:



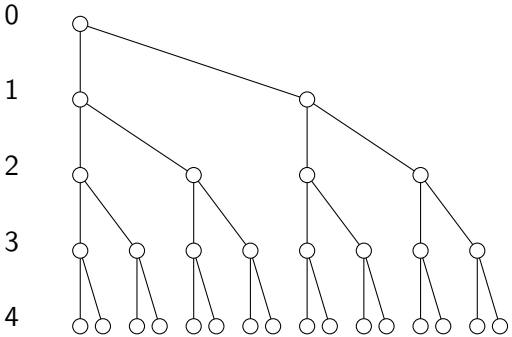
Bypass a): Modified Genuine Beams



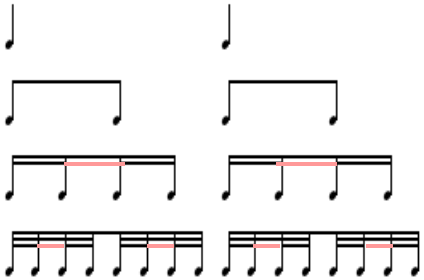
Modified genuine beams:



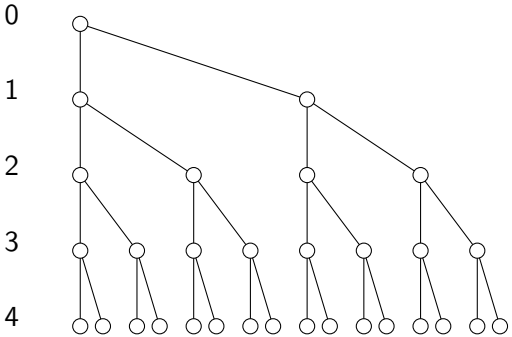
Bypass a): Modified Genuine Beams



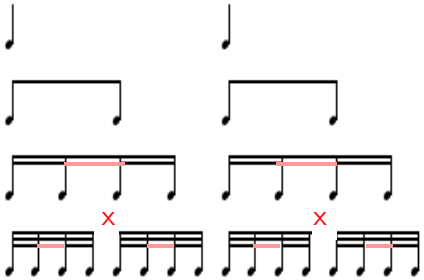
Modified genuine beams:



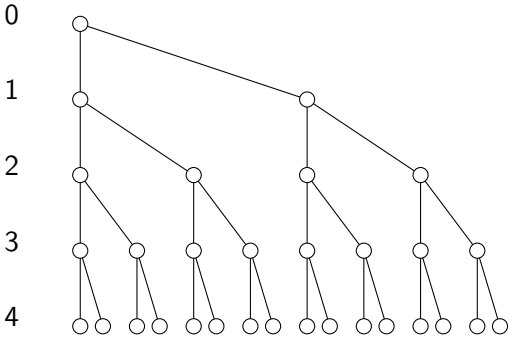
Bypass a): Modified Genuine Beams



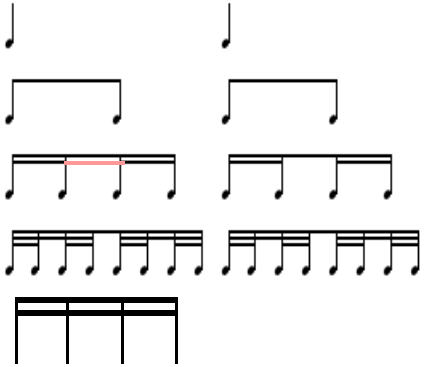
Modified genuine beams:



Bypass a): Modified Genuine Beams



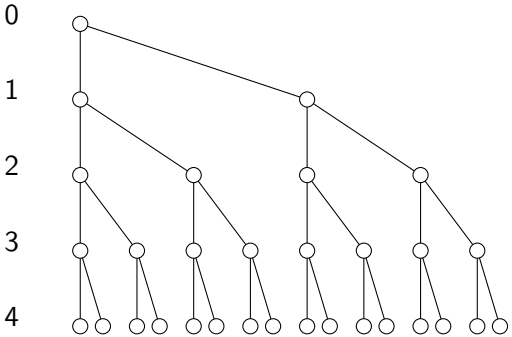
Modified genuine beams:



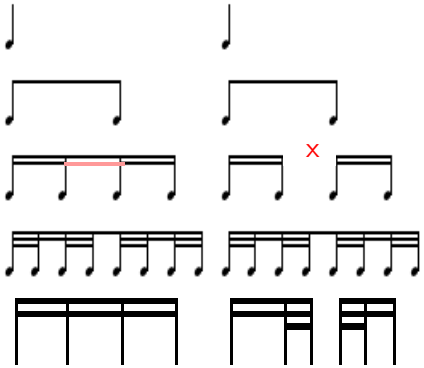
nota.trabes.alteraNatas : $\mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N})$
$= \{(2, +1, 2, 4)\}$

(means: at most 2 note heads on each side and 4 in total)

Bypass a): Modified Genuine Beams



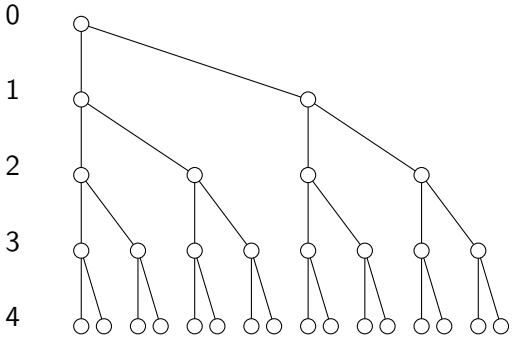
Modified genuine beams:



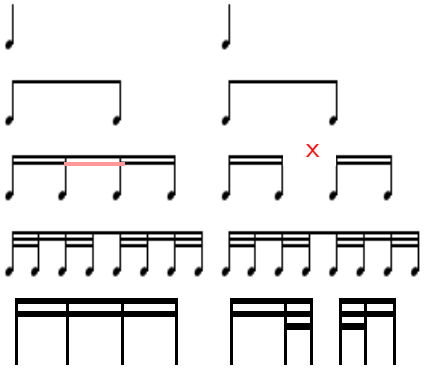
nota.trabes.alteraNatas : $\mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N})$
$= \{(2, +1, 2, 4), (2, -1, 3, 2)\}$

(means: at least 3 note heads on one side and 2 on both)

Bypass a): Modified Genuine Beams



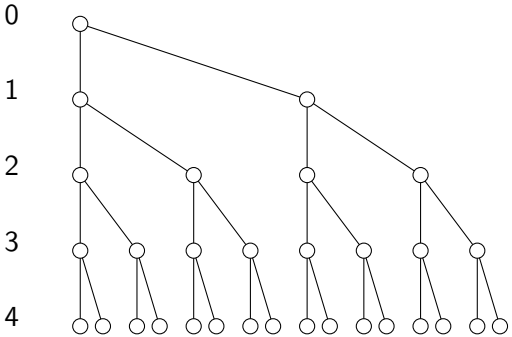
Modified genuine beams:



nota.trabes.alteraNatas : $\mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N})$
$= \{(2, +1, 2, 4), (2, -1, 3, 2)\}$

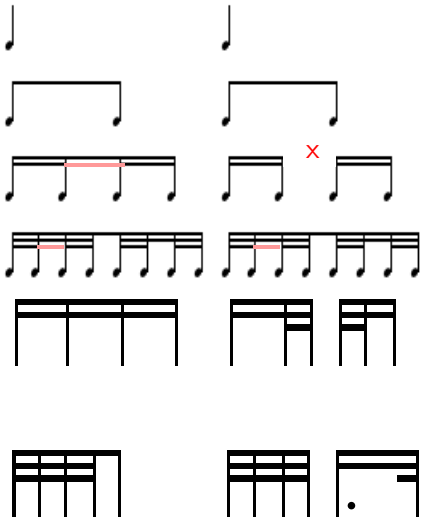
(means: at least 3 note heads on one side and 2 on both)

Bypass a): Modified Genuine Beams



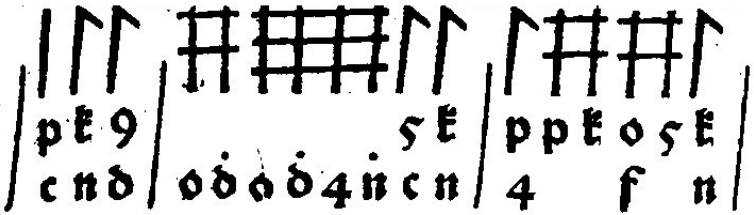
$$\textit{nota.trabes.alteraNatas} : \mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N}) = \{(2, +1, 2, 4), (2, -1, 3, 2), (3, +1, 2, 4)\}$$

Modified genuine beams:



Bypass a): Modified Genuine Beams

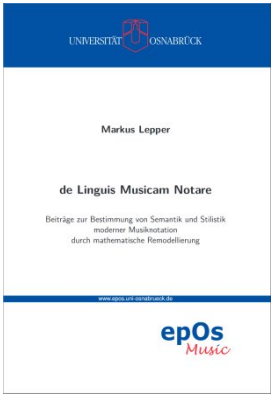
nota.trabes.alteraNatas $= \{(2, -1, 0, 0), (3, +1, \infty, \infty)\}$



(*Hans Neusidler: Ein Newgeordent Künstlich Lautenbuch* (Nuremberg, 1536))

Thanks

To get the whole picture you can



play with http://bandm.eu/downloads/DemoMetric.jar

contact post@mlepper.de

read de Linguis Musicam Notare, epOs, Osnabrück, 2021, EUR 44.90 / 24.90