

Beam It Up!

— A Classification Grid for Historic and Contemporary Practices of Beaming by Mathematical Re-Modelling

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Tenor 2023, Boston, May 17 2023



1 Context

- Genesis of the earliest beams
- Sort it out by mathematical re-modelling

2 Evaluation Pipeline for Beams

- Overview
- Metric Tree Spec and Rhythms
- Additional External Parameters
- Pitches and Vertical Positions
- Bypass a): Modified Genuine Beams

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(Hans Neusidler: *Ein Newgeordent Künstlich Lautenbuch* (Nuremberg, 1536),
urn:nbn:de:bvb:12-bsb00041542-7)

“Leiterlein” = “little ladder”

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Mathematical re-modelling =

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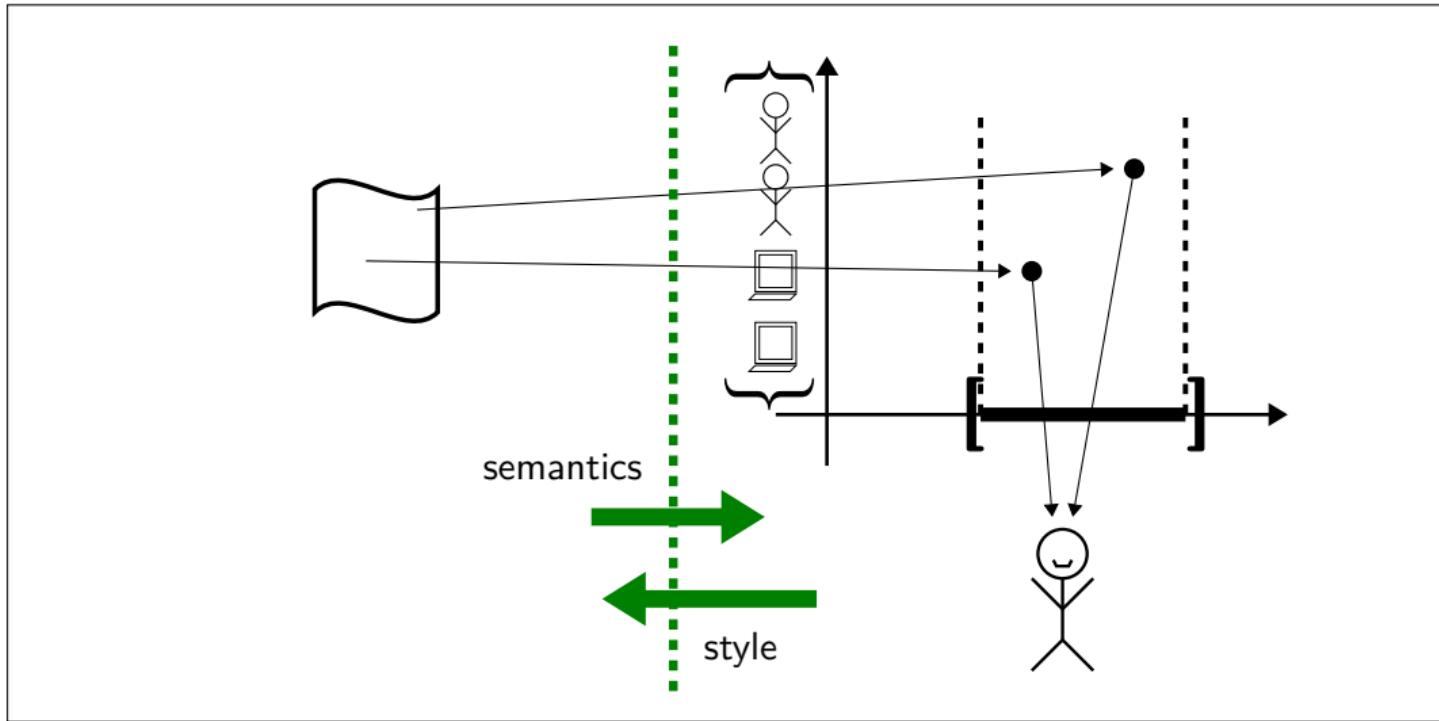
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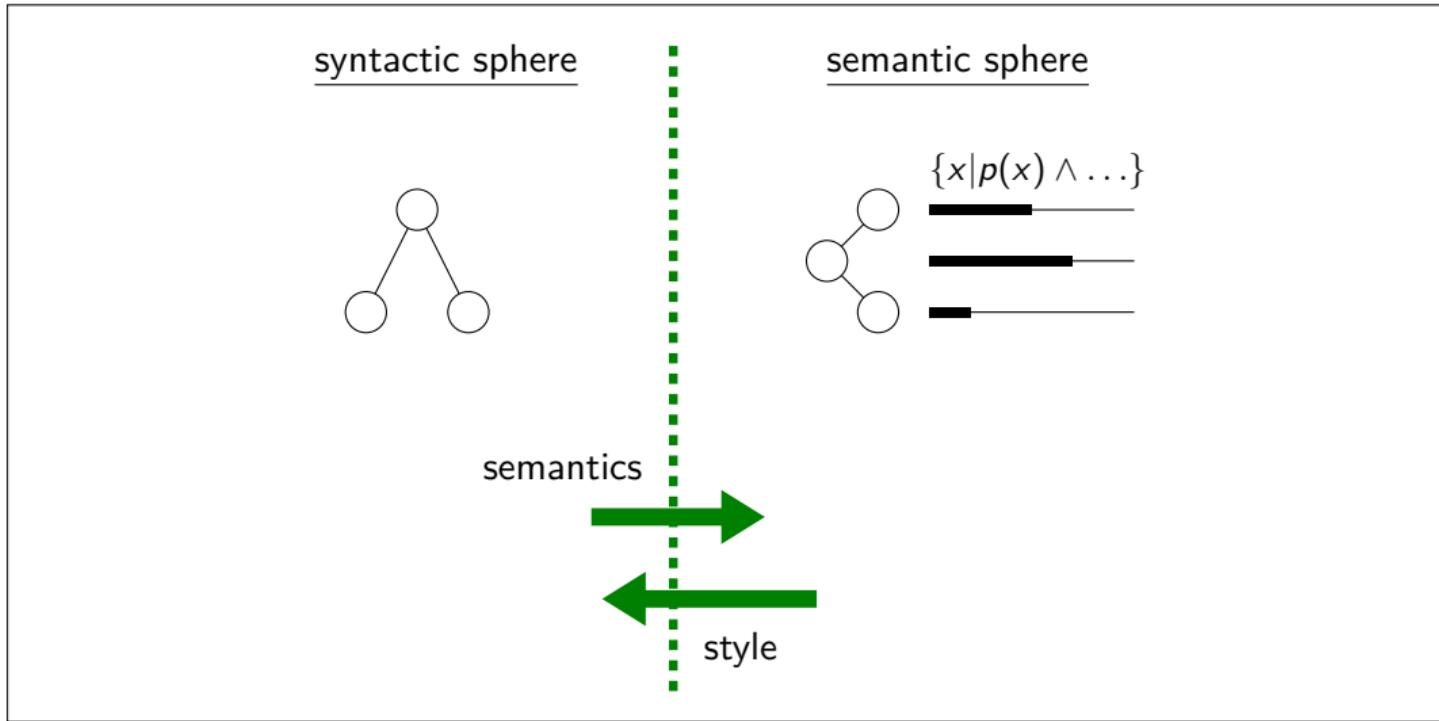
Mathematical re-modelling =

- Build a mathematical model which mimicks the symbol system defined by culture / history
- Build *variants / families* of models, identified by *parameter settings*
- With **no claim** for explanation, historic truth, adequateness,
but as **an offer** to domain experts for more precision in nomenclature.

Basic execution model of musical notation

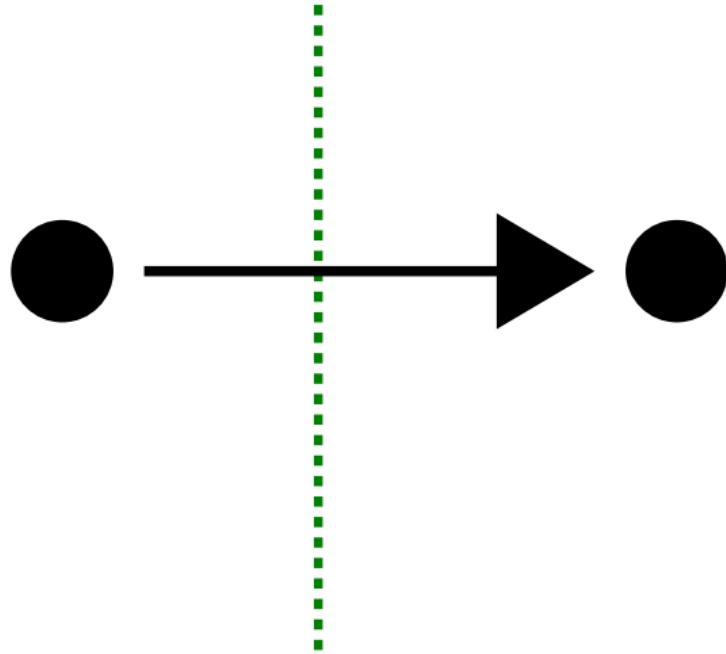


Basic execution model of musical notation



Basic execution model of musical notation

reading a score:



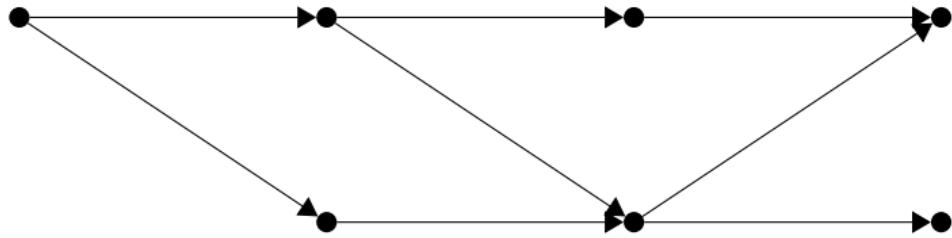
Basic execution model of musical notation

reading a score by evaluating a pipeline:



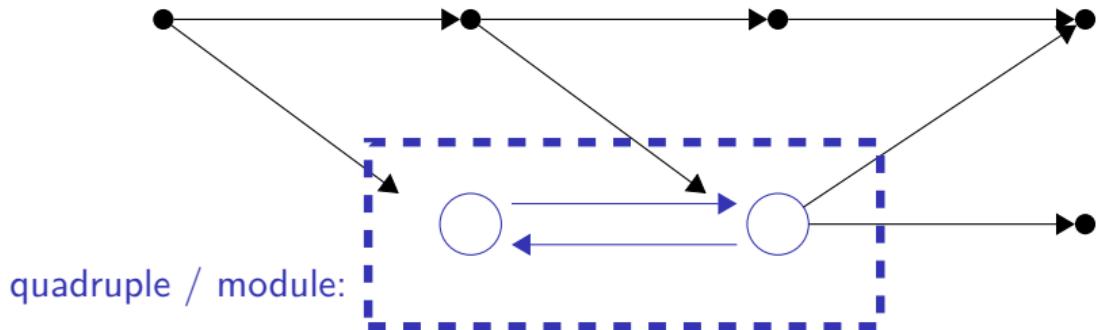
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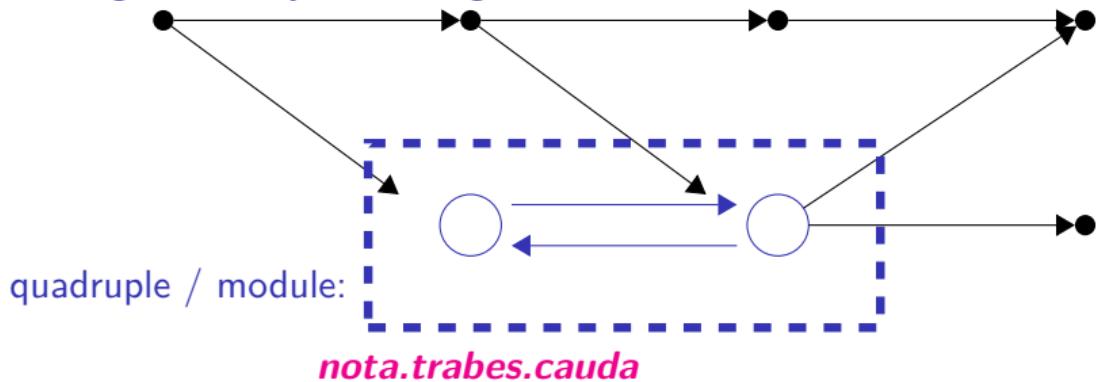
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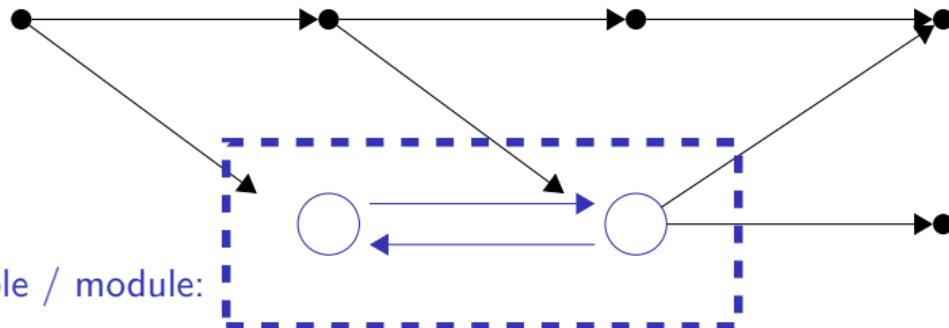
Basic execution model of musical notation

reading a score by evaluating a network:



Basic execution model of musical notation

reading a score by evaluating a network:



nota.trabes.cauda

names and types of parameters:

nota.trabes.cauda.minInterior: $\mathbb{Q}_{\geq 0}$

nota.trabes.cauda.min: $\mathbb{Q}_{\geq 0}$

⇒ model properties

de Linguis Musicam Notare (LMN)

- Complete theory of musical notation restricted to *conventional use of CWN*
- 834 properties (= modules and their parameters)
- 700 pages
- **open and extensible**
by “Z” refinement mechanism and hierarchical nomenclature
- These results on beams are such an extension to LMN: about 60 properties

1 Context

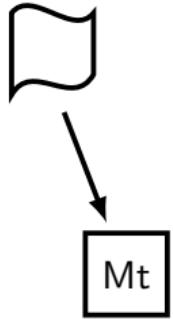
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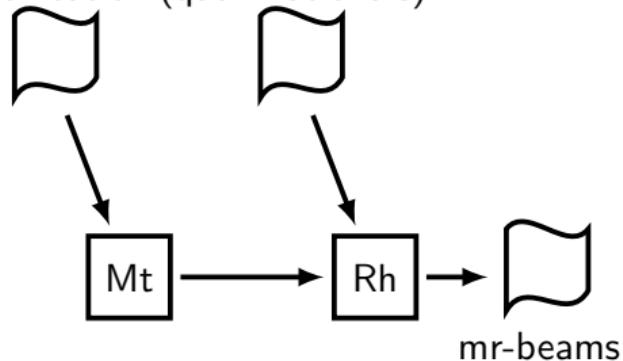
Beam pipeline overview:

Metric tree
specification

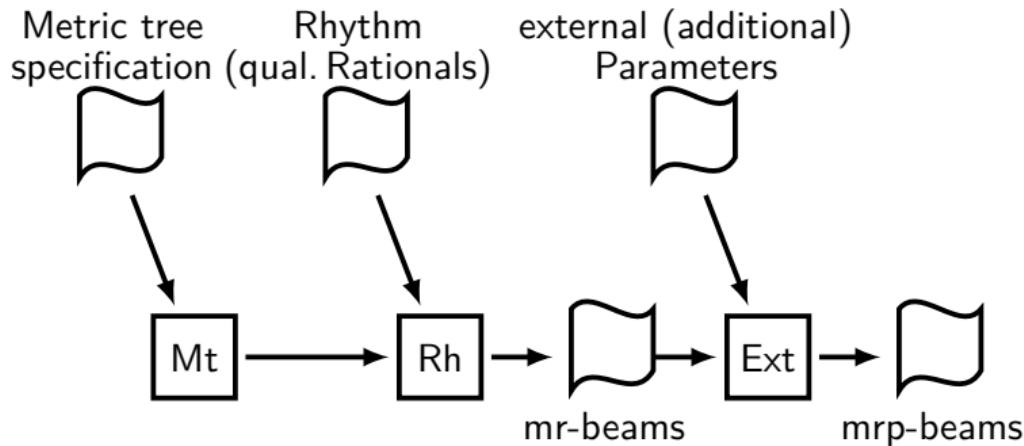


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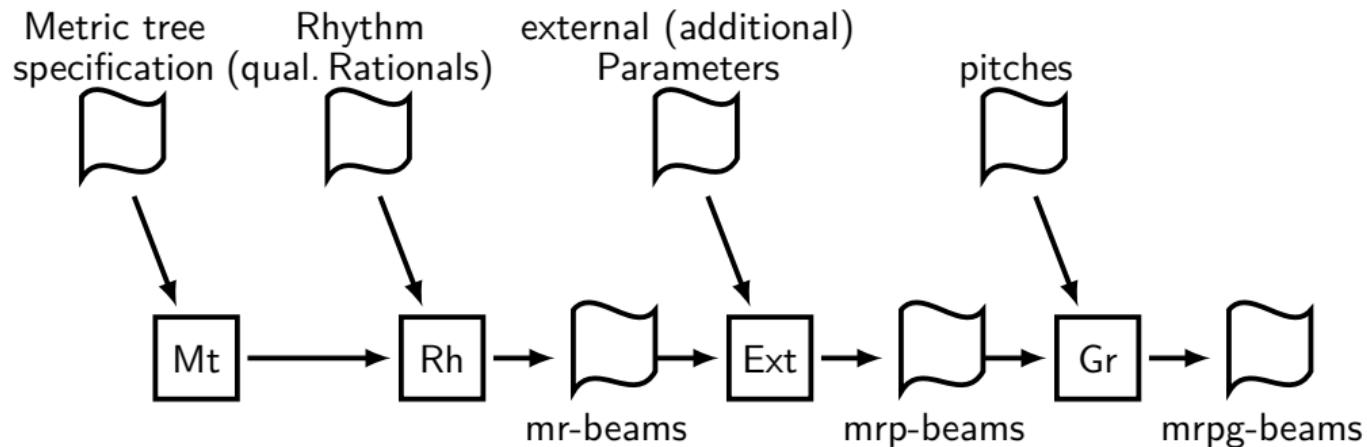
Metric tree Rhythm
specification (qual. Rationals)



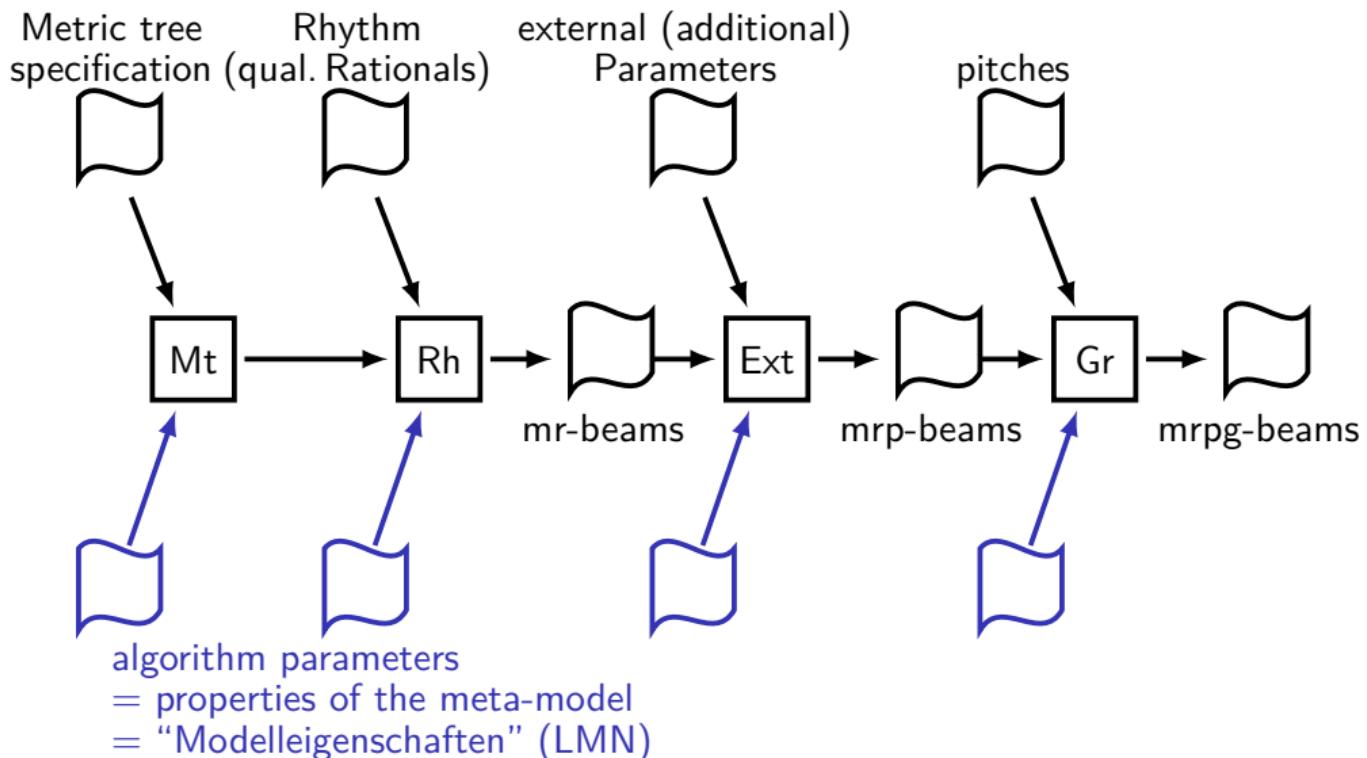
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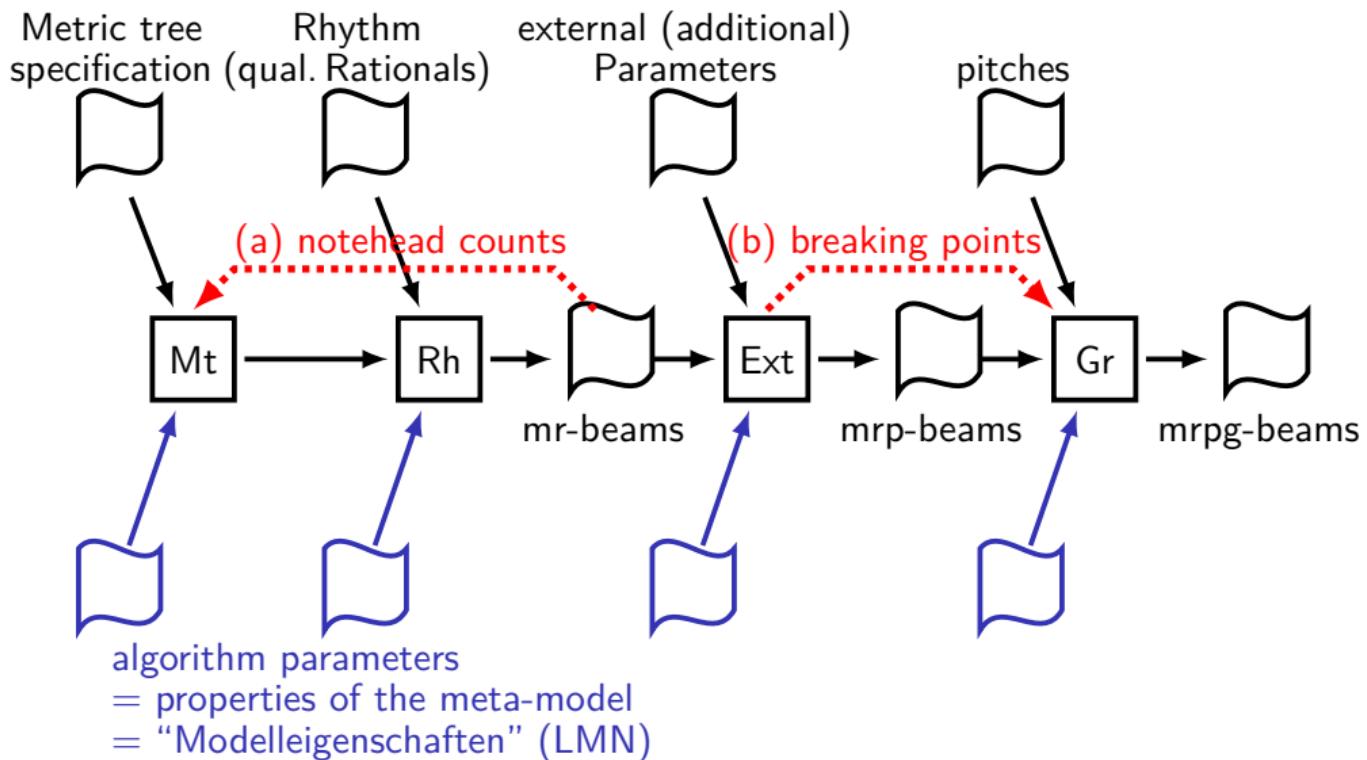
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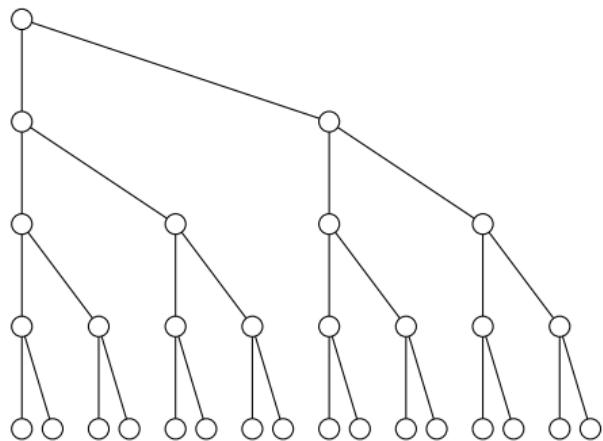


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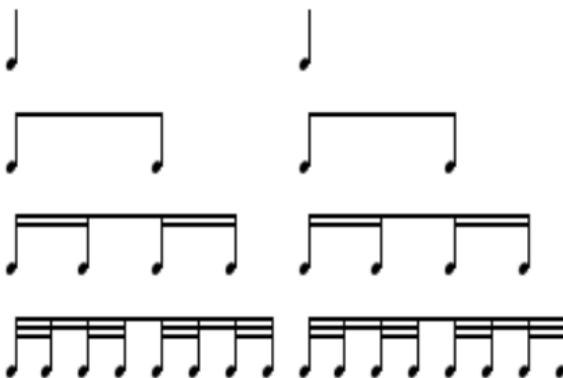


Metric Tree and Rhythm:

MTSpec = 1/2 (according to "metricSplit", Journal of Mathematics and Music, 2019)

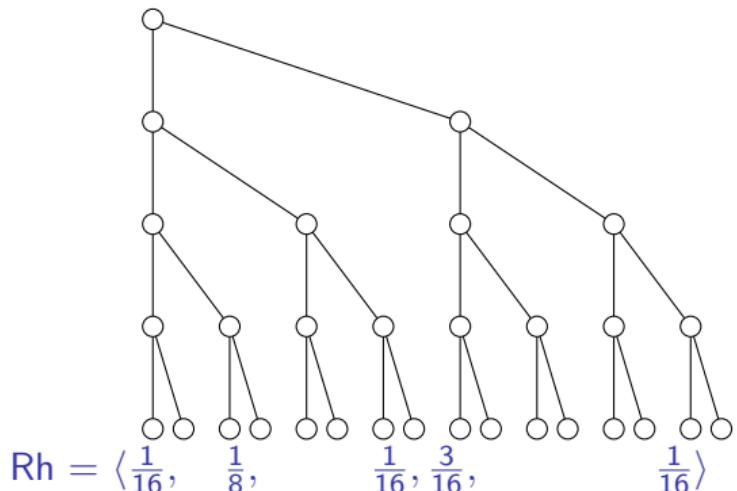


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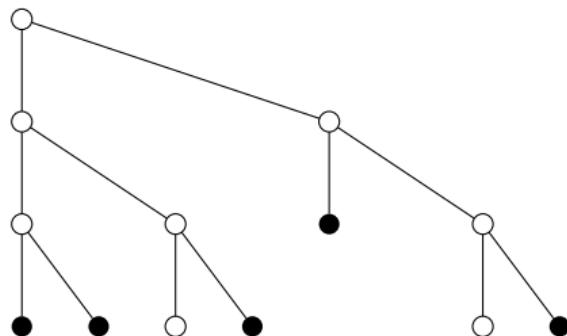
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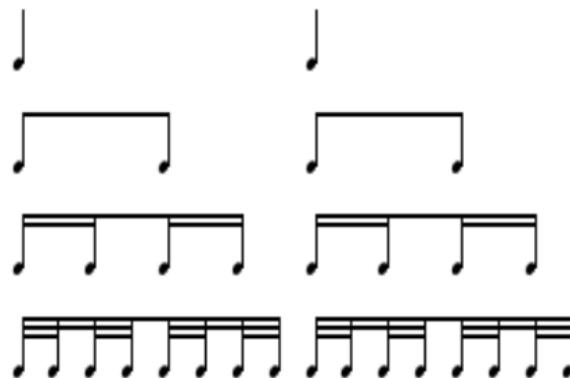
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Initial Coverage:



$$Rh = \left\langle \frac{1}{16}, \quad \frac{1}{8}, \quad \frac{1}{16}, \frac{3}{16}, \quad \frac{1}{16} \right\rangle$$

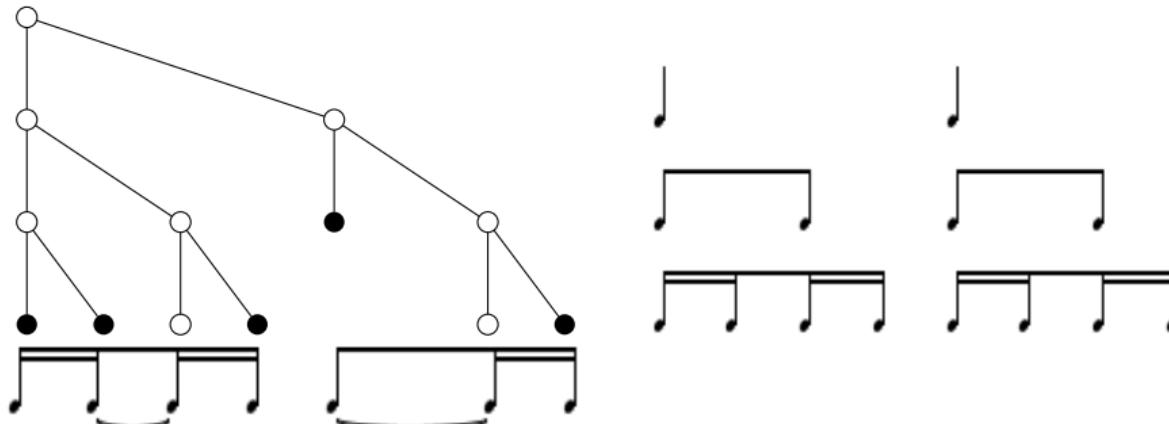
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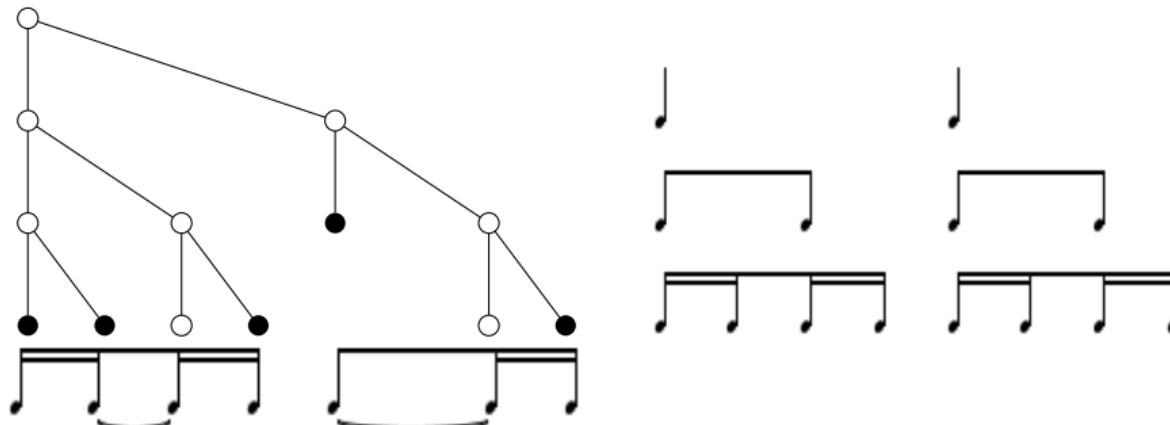


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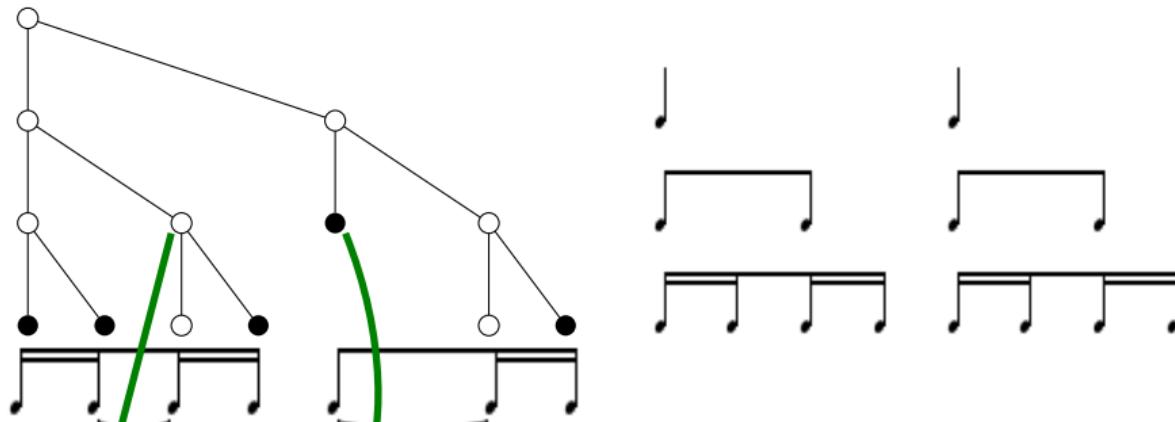
Merging Transformations MX-Y and MX-DP:



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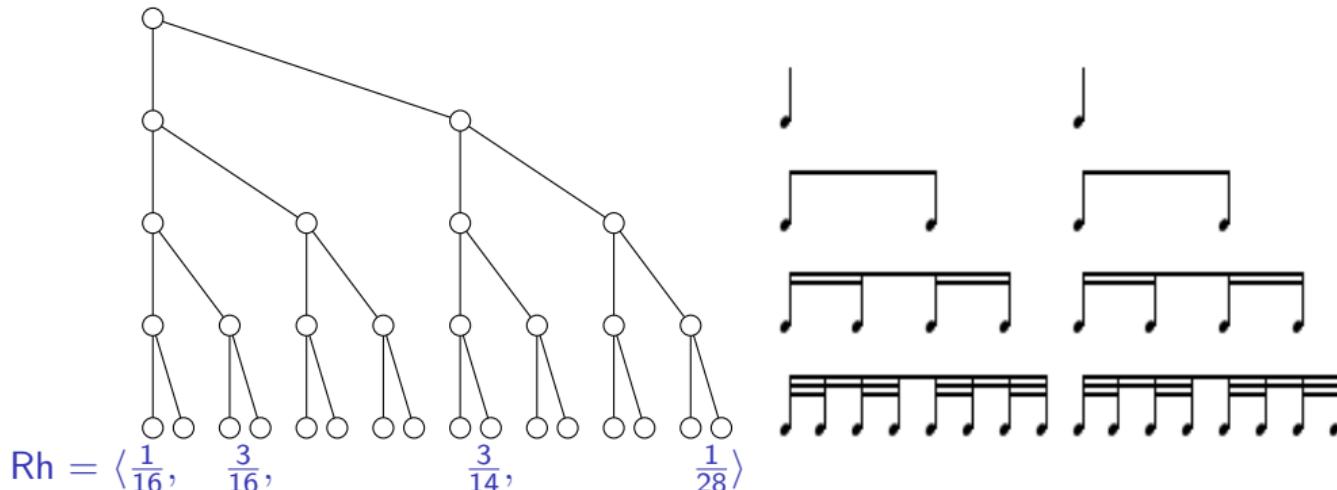


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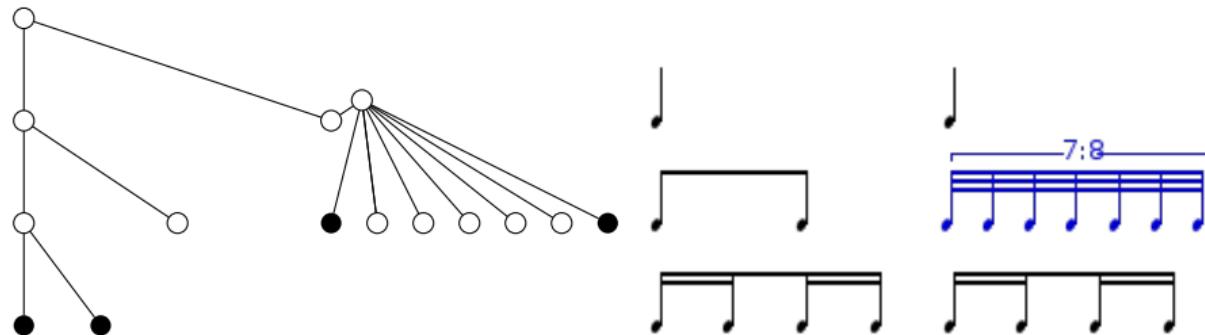
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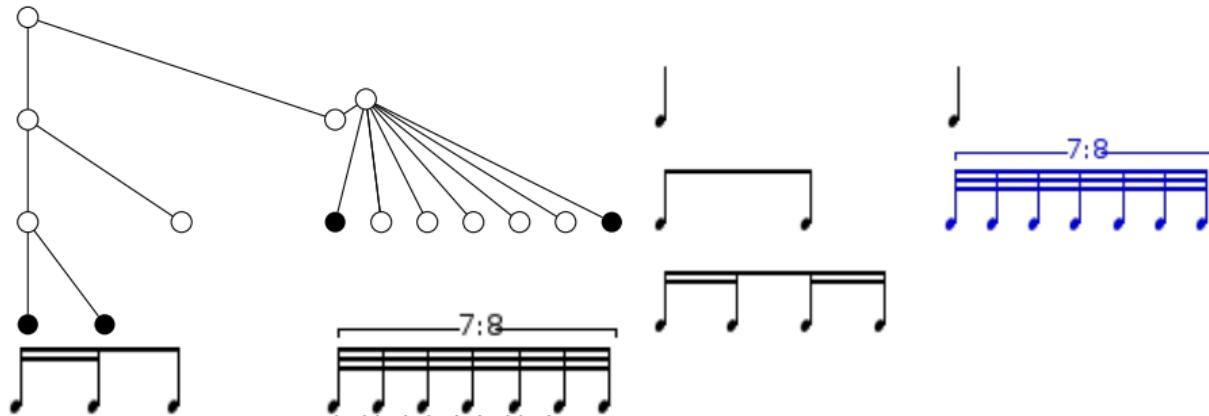


$$Rh = \left\langle \frac{1}{16}, \quad \frac{3}{16}, \quad \frac{3}{14}, \quad \frac{1}{28} \right\rangle$$

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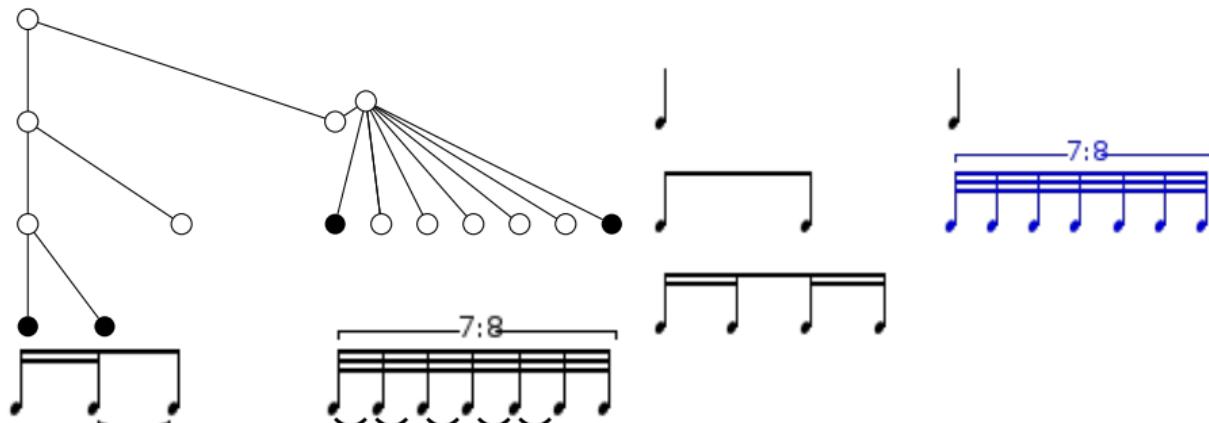


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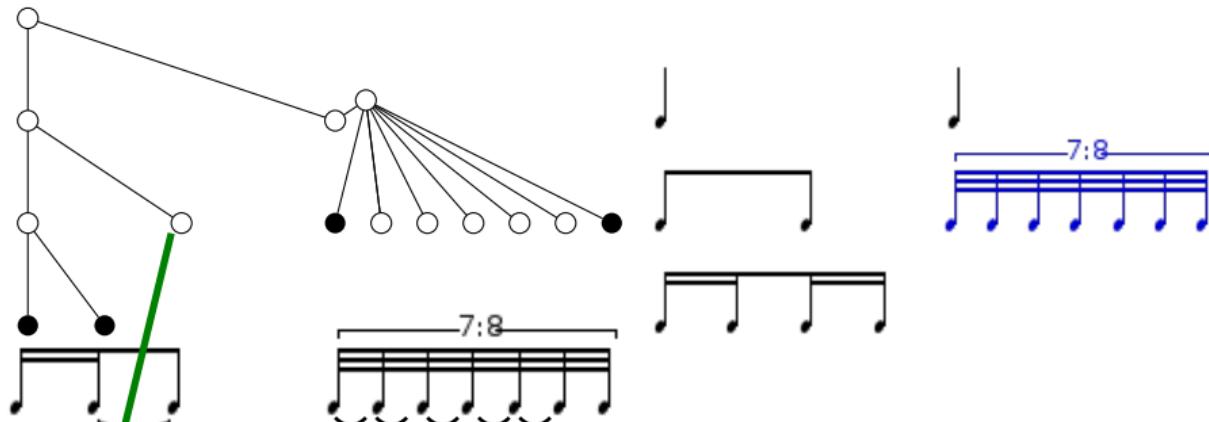
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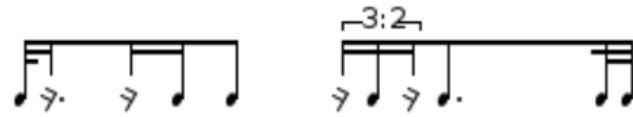


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Pauses and Generation and Cancellation of Beamlets:

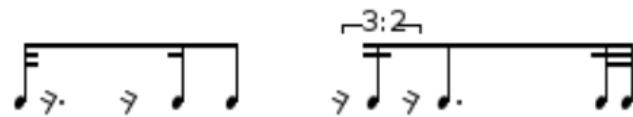
nota.trabes.superPausam.perCaudulum



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nota.trabes.superPausam.transiens

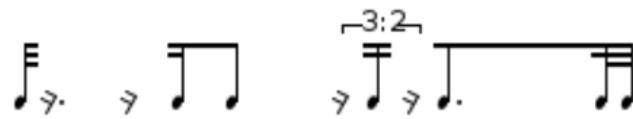


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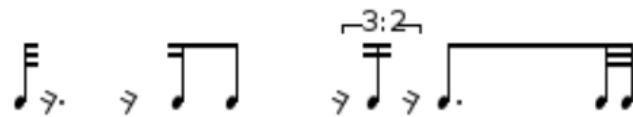
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nota.trabes.superPausam.ELIM-trabulaeOmnesContraTrabes



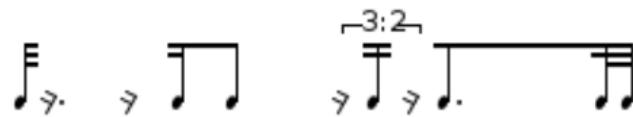
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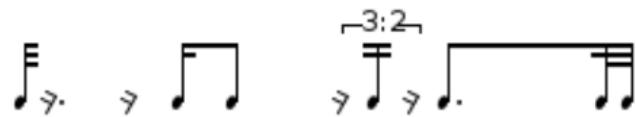
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nota.trabes.superPausam.ELIM-trabulaNonSubTrabem



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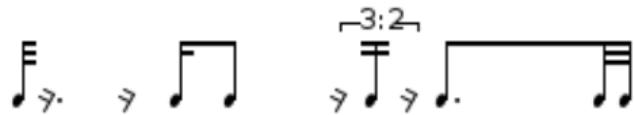
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..-ELIM-TrabulaeOmnesContraTrabes & ..ELIM-trabulaNonSubTrabem



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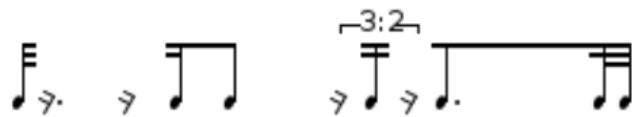
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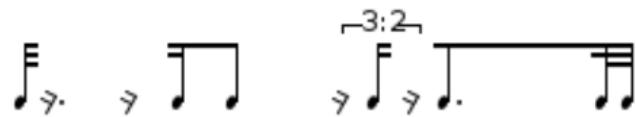
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..-ELIM-TrabulaeOmnesContraTrabes & ..ELIM-trabulaNonSubTrabem

nota.trabes.superPausam.ELIM-trabulaeOmnesContraldem(dexter)



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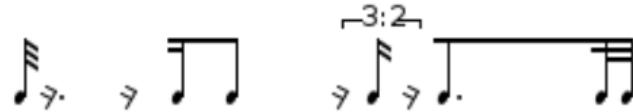
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nota.trabes.superPausam.ELIM-trabulaeOmnesContraldem(dexter)

nota.trabes.superPausam.ELIM-trabulaSola



Pauses and Generation and Cancellation of Beamlets:

$$(flags \mid leftLong, leftShort \mid rightLong, rightShort) : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N}$$

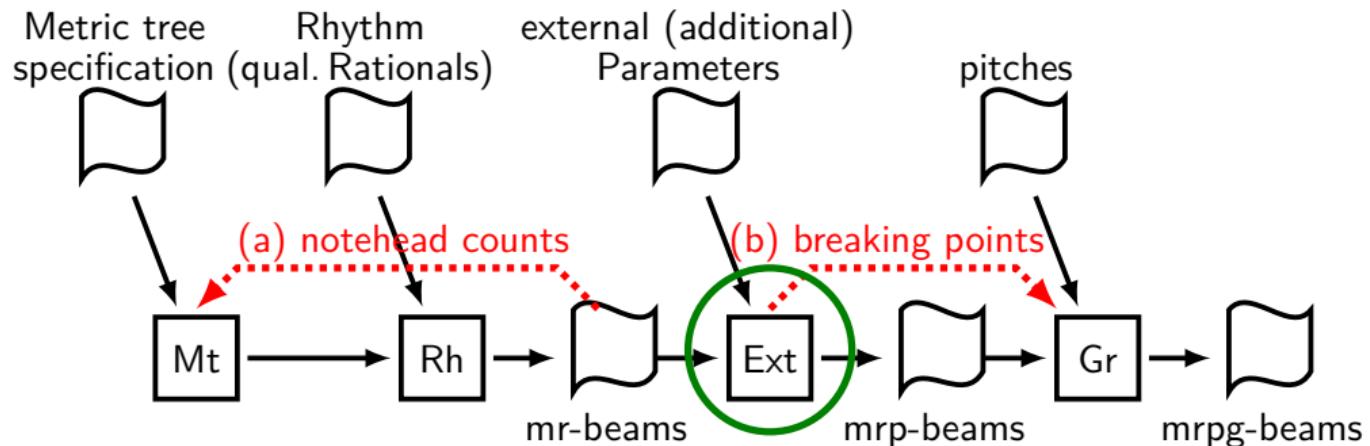
$$\frac{b > 0}{\begin{aligned} \text{ELIM-trabulaeContraldem}((0 \mid a, b \mid a, b), \text{dexter}) &= (0 \mid a, 0 \mid a, b) \\ \text{ELIM-trabulaeContraldem}((0 \mid a, b \mid a, b), \text{sinister}) &= (0 \mid a, b \mid a, 0) \end{aligned}}$$

$$\frac{b > 0 \quad a + b \leq c}{\begin{aligned} \text{ELIM-trabulaeOmnesContraTrabes}(0 \mid a, b \mid c, d) &= (0 \mid a, 0 \mid c, d) \\ \text{ELIM-trabulaeOmnesContraTrabes}(0 \mid c, d \mid a, b) &= (0 \mid c, d \mid a, 0) \end{aligned}}$$

$$\frac{b > 0 \quad c > 0 \quad x = \text{MAX}(d, b - c)}{\begin{aligned} \text{ELIM-trabulaNonSubTrabem}(0 \mid 0, b \mid c, d) &= (0 \mid 0, 0 \mid c, x) \\ \text{ELIM-trabulaNonSubTrabem}(0 \mid c, d \mid 0, b) &= (0 \mid c, x \mid 0, 0) \end{aligned}}$$

$$\frac{b + d > 0}{\text{ELIM-trabulaSola}(0 \mid 0, b \mid 0, d) = (\text{MAX}(b, d) \mid 0, 0 \mid 0, 0)}$$

Additional External Parameters



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nota.trabes.cumVerborum.syllabis/nominibus/lineis[Trancón]

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nota.trabes.separate.cumMelo/cumLegato



Additional External Parameters

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nota.trabes.separate.cumMelo/cumLegato

nota.trabes.cumPositioneManus (\in *ligans!*)



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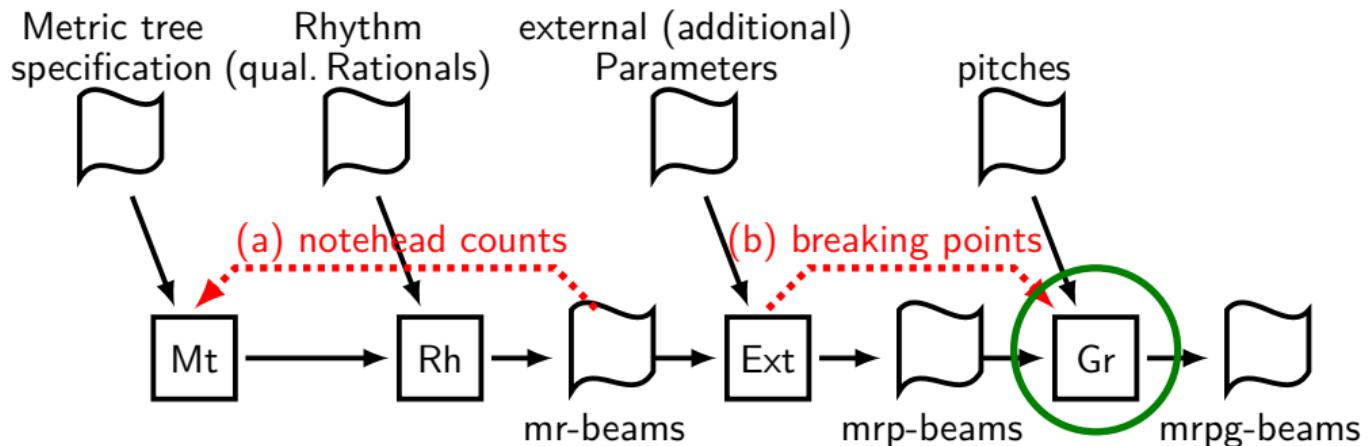


nota.trabes.accelerans : $(\mathbb{Q} \times \mathbb{N}_{>0} \times \mathbb{Q})^*$

= $\langle (0, 30, 1/1)(1/4, 90, 3/2), (1/4, 60, 1/1), (1/2, 60, 1/1), (1/2, 30, 1/2), (1/1, 120, 3/2) \rangle$



Pitches and Vertical Positions



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Vertical arrangement, according to pitches and graphical constraints, may . . .

- be trivial

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Vertical arrangement, according to pitches and graphical constraints, may . . .

- be trivial
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- possibly controlled by a **by-pass b**): penalties (“Sollbruchstellen”):



(a) Flute

(b) Piano (*cumPositioneManus*)

Pitches and Vertical Positions

Vertical arrangement, according to pitches and graphical constraints, may ...

- be trivial
 - or need breaking into sub-aggregates
 - possibly controlled by a **by-pass b**): penalties (“Sollbruchstellen”):

A musical score page from Beethoven's Violin Concerto in D major, Op. 61, 1st movement. The score is for orchestra and includes parts for Violin, Viola, Cello, Double Bass, Flute, Clarinet, Bassoon, and Trombones. The page shows measures 11 and 12. Measure 11 starts with a forte dynamic in common time. Measure 12 begins with a piano dynamic.

(a) Flute

(b) Piano (*cumPositioneManus*)

- or require to switch the beam side, forming a “knee” (*genus.inLineae/interLineas*):

The diagram illustrates six pairs of musical notes connected by vertical lines, representing different note pairings:

- $(<,<)$
- $(<,=)$
- $(<,>)$ (highlighted in red)
- $(<,>>)$

nota.trabes.genuParadoxum
(stands against . . . *inclinationSignificans*)

Pitches and Vertical Positions

Vertical arrangement, according to pitches and graphical constraints, may ...

- be trivial
 - or need breaking into sub-aggregates
 - possibly controlled by a **by-pass b**): penalties (“Sollbruchstellen”)

A musical staff in G major (one sharp) and common time. The first measure contains a half note followed by a sixteenth-note cluster (two eighth notes and a sixteenth note). The second measure contains a sixteenth-note cluster (two eighth notes and a sixteenth note) followed by a half note.

(a) Flute

(b) Piano (*cumPositioneManus*)

- or require to switch the beam side, forming a “knee” (*genus.inLineae/interLineas*):

The diagram illustrates six pairs of musical notes connected by vertical lines, representing different note pairings:

- $(<,<)$: Two eighth notes connected by a single vertical line.
- $(<,=)$: An eighth note and a sixteenth note connected by a single vertical line.
- $(<,>)$: An eighth note and a quarter note connected by a single vertical line.
- $(<,>>)$: An eighth note and a half note connected by a single vertical line.
- $(=,>)$: A sixteenth note and a quarter note connected by a single vertical line.
- $(>,>)$: Two quarter notes connected by a single vertical line.

nota.trabes.genuParadoxum
(stands against . . . *inclinationSignificans*)

A musical score fragment on a staff. It starts with a treble clef, followed by two sharp signs indicating key signature. A fermata (a curved line over a note) is placed above a note. A grace note is shown as a small eighth note with a diagonal line through it.

Pitches and Vertical Positions

Vertical arrangement, according to pitches and graphical constraints, may . . .

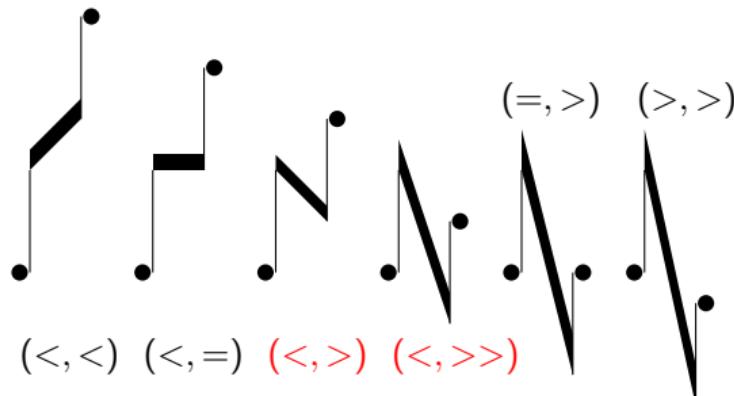
- be trivial
- or need breaking into sub-aggregates
- possibly controlled by a **by-pass b**): penalties (“Sollbruchstellen”):



(a) Flute

(b) Piano (*cumPositioneManus*)

- or require to switch the beam side, forming a “knee” (*genus.inLineae/interLineas*):



nota.trabes.numerusGenuum: \mathbb{N}_0

nota.trabes.numerusInGenii: \mathbb{N}_1

nota.trabes.numerusInterGenibus: \mathbb{N}_1

nota.trabes.caudaVersusCaudam: bool

nota.trabes.genuParadoxum

(stands against . . . *inclinationSignificans*)

More Details on Knees:

nota.trabes.summaeInGenu



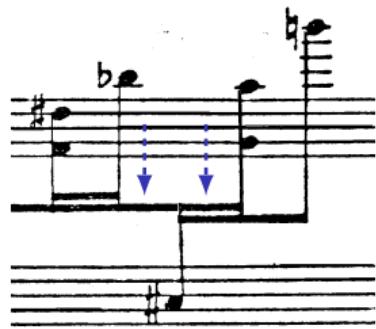
nota.trabes.inGenuCumPluribus



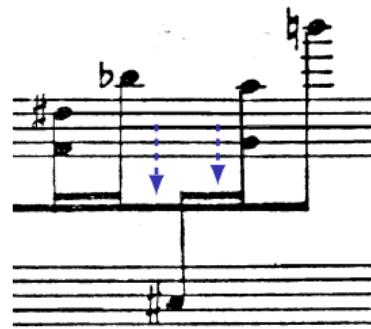
(Original Xenakis, Herma)

More Details on Knees:

nota.trabes.summaeInGenu



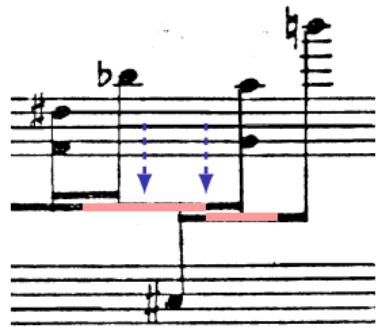
nota.trabes.inGenuCumPluribus



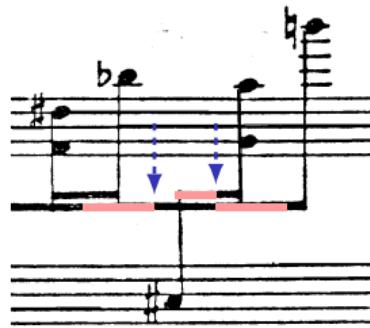
(Original Xenakis, Herma)

More Details on Knees:

nota.trabes.summaInGenu



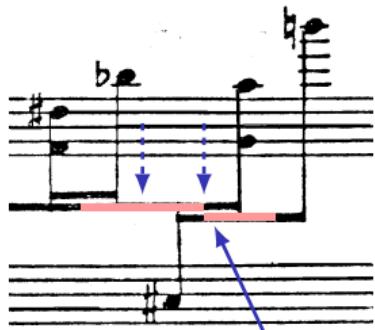
nota.trabes.inGenuCumPluribus



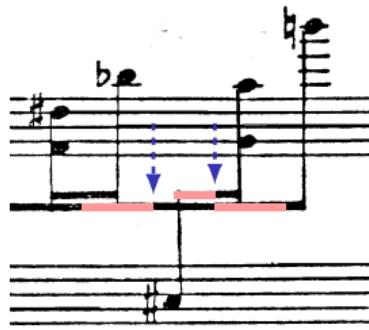
(Original Xenakis, Herma)

More Details on Knees:

nota.trabes.summaeInGenu



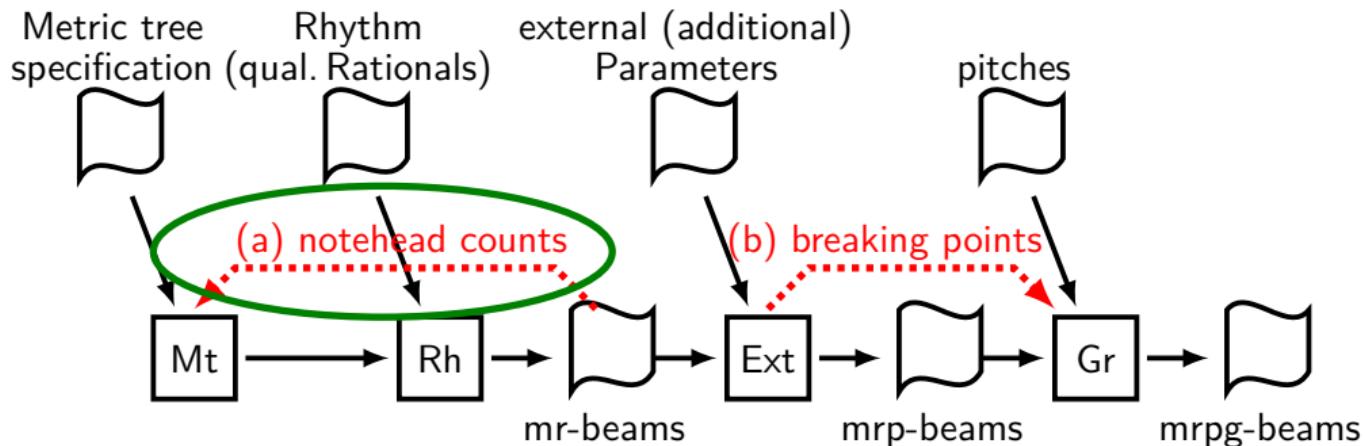
nota.trabes.inGenuCumPluribus



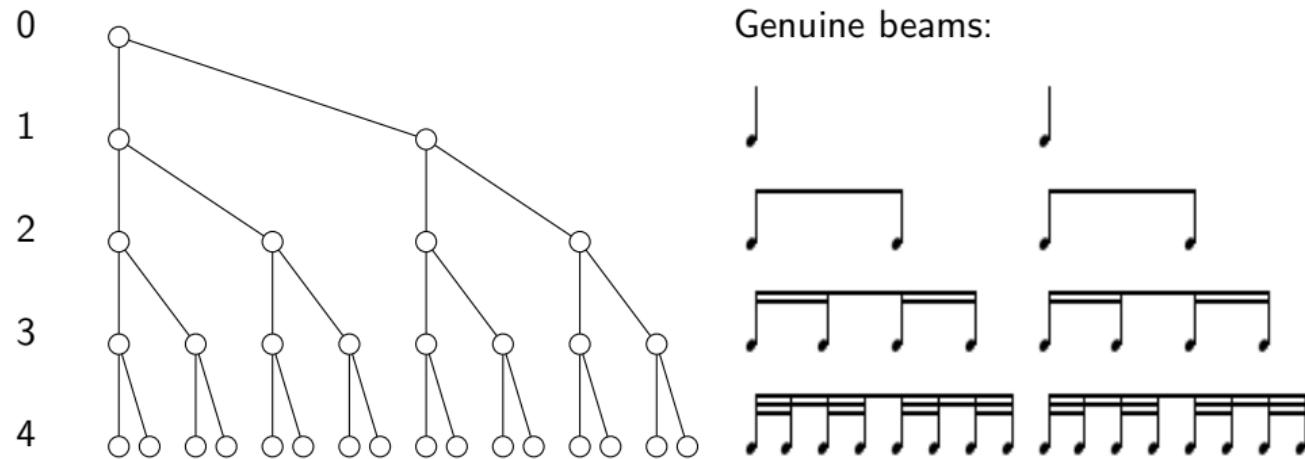
(Original Xenakis, Herma)

Change of meaning is meaningless / non-deterministic determinedness
= *infra.nonDeterminataSedDeterminans[Pepper]*

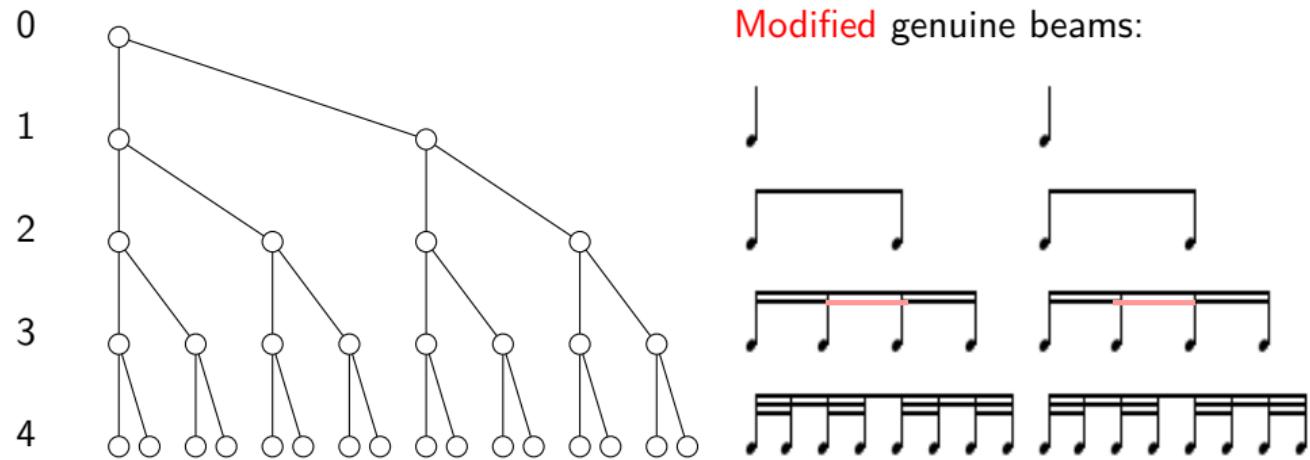
Bypass a): Modified Genuine Beams



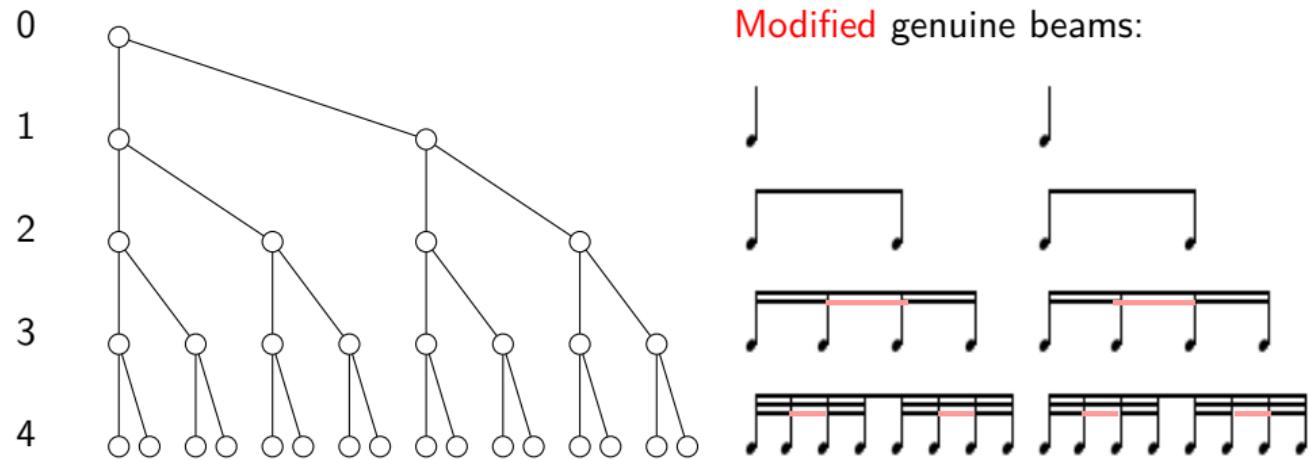
Bypass a): Modified Genuine Beams



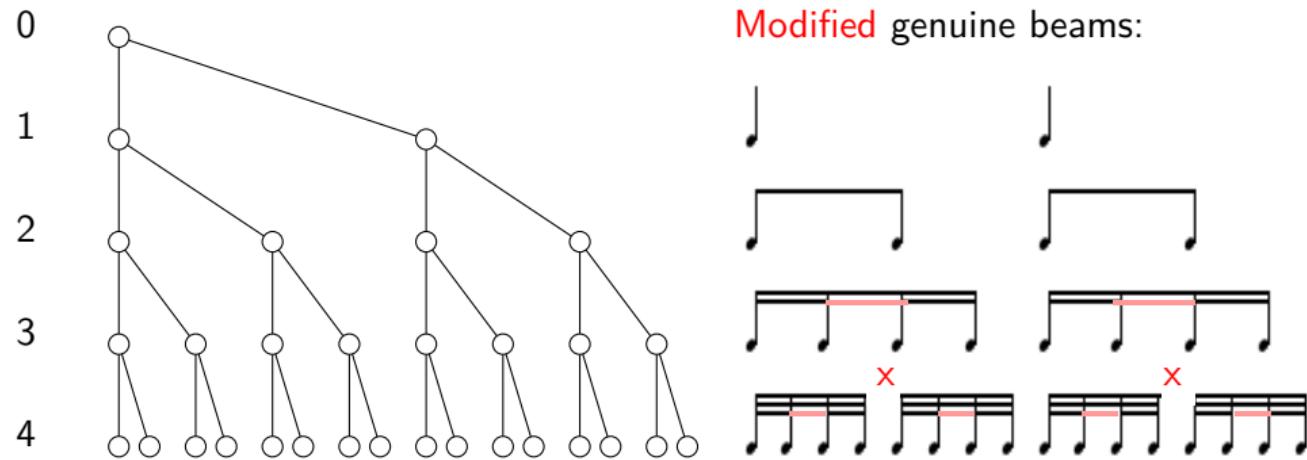
Bypass a): Modified Genuine Beams



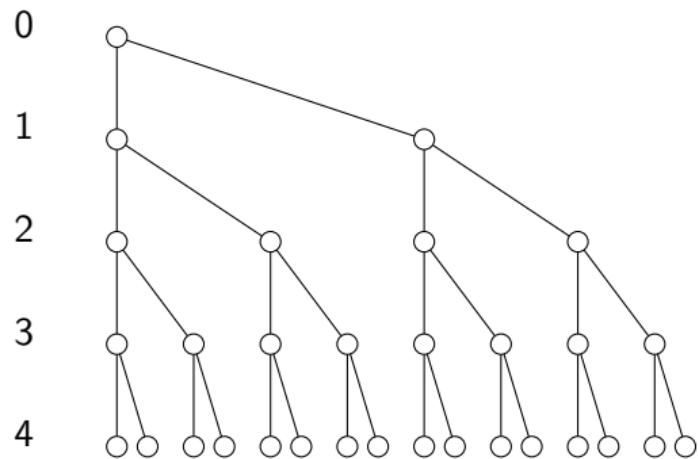
Bypass a): Modified Genuine Beams



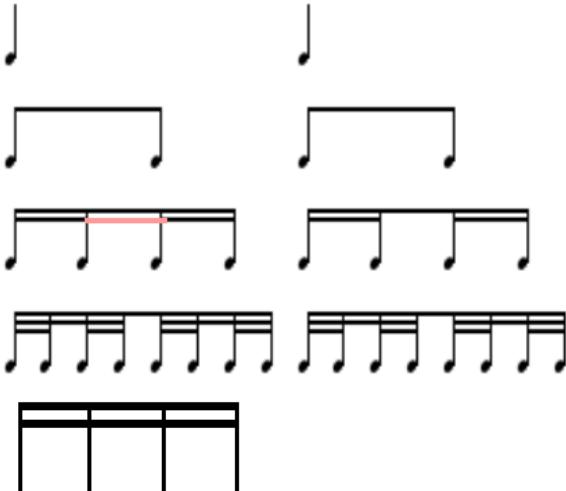
Bypass a): Modified Genuine Beams



Bypass a): Modified Genuine Beams



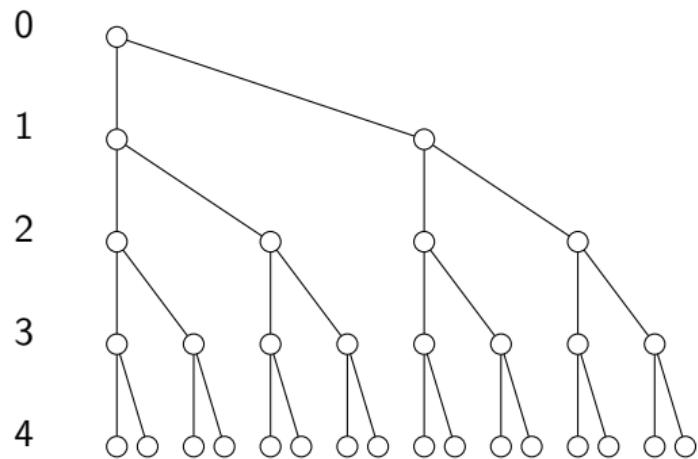
Modified genuine beams:



$$\begin{aligned} \text{nota}.trabes.alteraNatas : & \mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N}) \\ = & \{(2, +1, 2, 4)\} \end{aligned}$$

(means: at most 2 note heads on each side and 4 in total)

Bypass a): Modified Genuine Beams



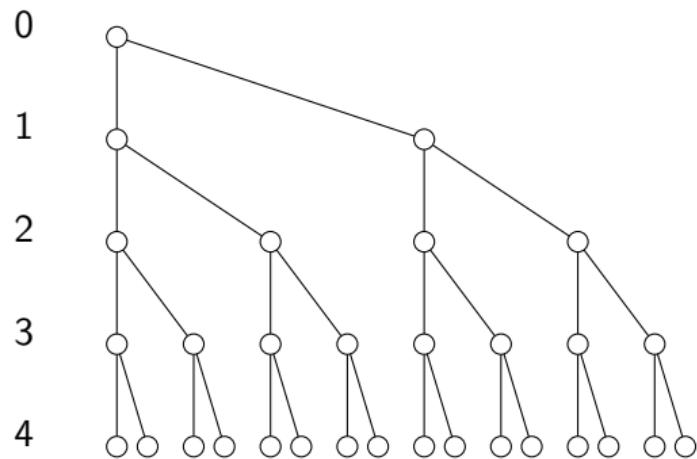
Modified genuine beams:



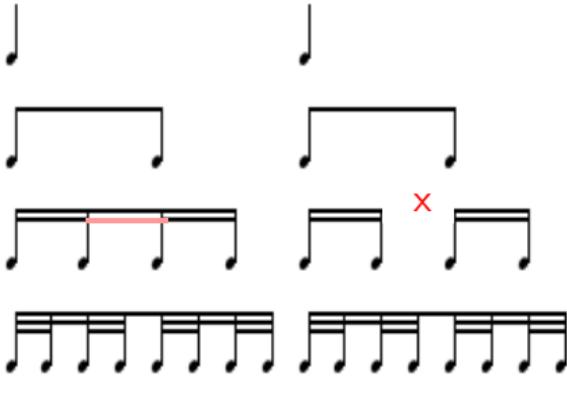
$$\begin{aligned} \text{nota}.trabes.alteraNatas : & \mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N}) \\ = & \{(2, +1, 2, 4), (2, -1, 3, 2)\} \end{aligned}$$

(means: at least 3 note heads on one side and 2 on both)

Bypass a): Modified Genuine Beams



Modified genuine beams:

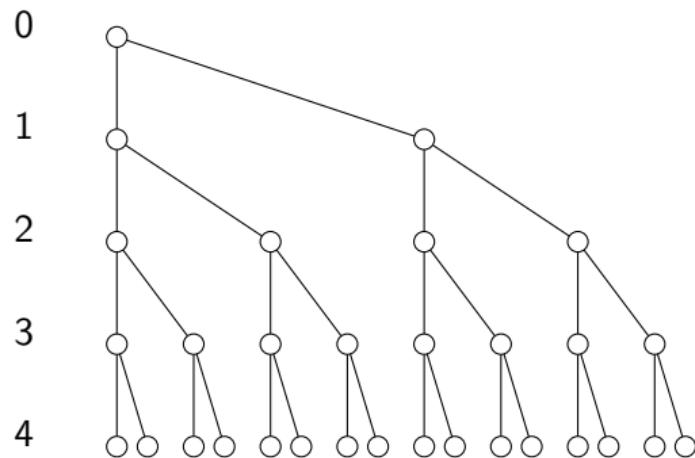


$$\begin{aligned} \text{nota}.trabes.alteraNatas &: \mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N}) \\ &= \{(2, +1, 2, 4), (2, -1, 3, 2)\} \end{aligned}$$

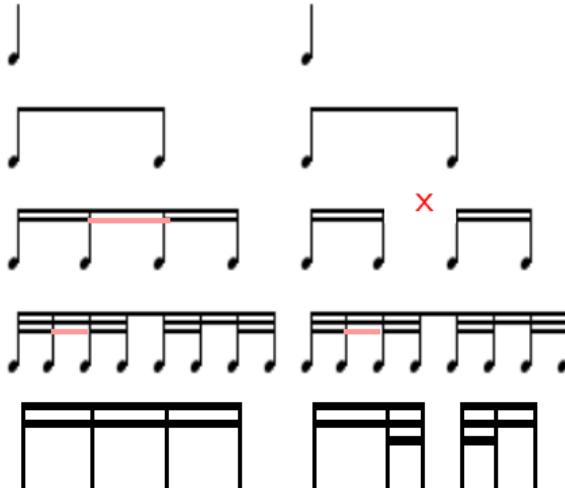
(means: at least 3 note heads on one side and 2 on both)



Bypass a): Modified Genuine Beams



Modified genuine beams:



nota.trabes.alteraNatas : $\mathbb{P}(\mathbb{N} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N})$
 $= \{(2, +1, 2, 4), (2, -1, 3, 2), (3, +1, 2, 4)\}$



Bypass a): Modified Genuine Beams

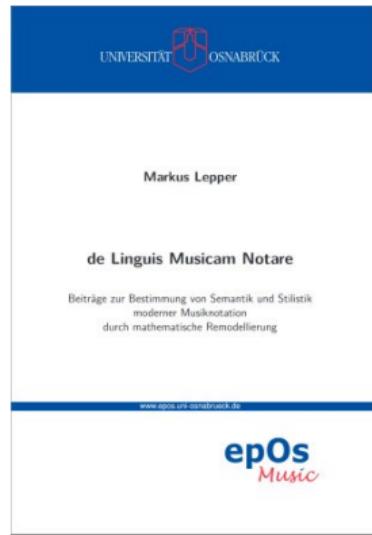
nota.trabes.alteraNatas = $\{(2, -1, 0, 0), (3, +1, \infty, \infty)\}$



(Hans Neusidler: *Ein Newgeordent Künstlich Lautenbuch* (Nuremberg, 1536))

Thanks

To get the whole picture you can



play with <http://bandm.eu/downloads/DemoMetric.jar>

contact post@mlepper.de

read [de Linguis Musicam Notare, epOs, Osnabrück, 2021, EUR 44.90 / 24.90](#)